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Teaching for Transfer of Training in Mathematics*

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1. *Introduction. Transfer of training* has been much discussed. The usual discussions are concerned with the transfer of factual knowledge, or of skills, or of types of abilities, to other fields than that in which they were acquired, or to life. Thus one enquires whether a training in logical thinking (gained, say, in the study of *plane geometry*) "transfers" to such fields as *law*, or *economics*, or *diplomacy*.

There are some experiments of great value. Investigations may be defective, however, even when they are sincere. Thus, one reason why students who have taken considerable *mathematics* are good in *law* is that the general intelligence ("I.Q.") is probably higher in the group that chooses to take mathematics than it is in the group that avoids mathematics.

Such difficulties confuse the issue, and they give an appearance of transfer of training that may be misleading. For these reasons, and analogous ones, claims have been made for mathematics as a preparation for law, for example, which are probably in excess of the realities. Certain other reasons that I shall mention a little later lead me to suspect that claims thus made are *far in excess* of the realities.

Experiments conducted on animals are more reliable in that the check groups can be made comparable with those subjected to a given training, and because the experimentation is not so dangerous as it might be with human subjects. Such experiments, begun by Thorndike of this institution, and carried on by many other investigators, have led to disbelief in transfer of training, even in comparitively related fields. So striking were these results that many were led to make statements regarding *failure* of transfer that are no less extreme (and no less faulty) than are the claims *for* transfer which I have mentioned above.

The total result is not entirely clear. At present, no two psychologists would agree with any precision as to the extent to which we may expect transfer. About what can be said is that there is *some* transfer of training in one activity to some closely related activities, but that there is very much less transfer than had been supposed, say, in 1900, and far less than is supposed still by many teachers of mathematics. That these statements are vague and are open to subjective interpretations that vary tremendously, is not only admitted:—it is in fact a true mark of the uncertainty that exists concerning this whole matter.

Nor does it simplify anything to say

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that transfer will exist whenever there are "identical elements" (whatever that may mean) in the subject or topic taught and in the thing to which it is applied. I shall have occasion to mention to you instances of *complete* (not *partial*) *identity* in which transfer *does not* occur.

2. *Faculties of the Mind.* An older psychology had seemed to present a sound basis for transfer of training. Indeed, the experiments and investigations on transfer have been one primary reason for the abandonment of that theory of psychology. I refer, of course, to the so-called "faculty-psychology," which attempted to analyze the mind into *faculties*, such as *memory*, *will*, *reason*, and other neat compartments. It turns out that a brain, whether it be of an animal or of a man, is not so simple a thing as was supposed in such a theory. Thus *memory* does not seem to be a single "faculty": it is probable that many of you can remember, as I can, *faces*, but not *names*: you may be able to memorize poetry, but not telephone numbers, or nonsense syllables; or conversely.

It is so with each of the supposed "faculties." There is no single memory ability. There is no single will-power. There is no single reasoning faculty. It is even hinted that there is no single intelligence-quotient, though many who condemn loudly the faculty-psychology of the past maintain equally stoutly the universal validity of one single "intelligence."

If there are no separate unit "faculties," the ability to train each such "faculty" by appropriate exercises, so that it shall function accurately in all possible situations, is clearly illusory. Thus the memorization of poetry, or of nonsense syllables, does not contribute toward ability to remember faces or names or telephone numbers. Memorization of mathematical formulas contributes toward ability to remember . . . just such formulas!

3. *Transfer inside of Mathematics.* If we *question* transfer from geometry to law, or if we *believe* strongly in it, experimentation on animals is impossible, and experiment on human beings is too dangerous, and is likely to be illusory. It has long ago occurred to me that there are questions of transfer that are easier to judge and simpler to control.

These are the questions of transfer *within the same subject*, for example, within mathematics itself. Such instances exist in large numbers, and the facts about them are relatively easy to discover. They offer also more intimate cases of "transfer," since the subjects concerned have a high degree of identity.

There has been comparitively little study of these matters, and there is not even a clear recognition that the instances I shall mentioned are included under the general subject of transfer. I shall mention some specific instances, and I shall try to make clear that the questions that arise are exactly the questions of transfer of training, though the subjects are very closely related. This fact makes *absence of transfer* or *difficulty of transfer* all the more striking.

Thus, instead of asking whether the logical training secured in a study of geometry is transferred to logical argument in such a subject as *law*, I may raise the question as to whether or not logical training in *geometry* is transferred to logical training in *algebra*.

In geometry, many teachers, perhaps many of you, hold that it is necessary for any intelligent person to prove his statements by deduction from a system of axioms, and that failure to do so is a mark of great mental inferiority. We even hold, sometimes, that this habit of logical deduction should affect the student's *whole life*, and that he should be trained by this study of geometry to proceed similarly in all other subjects and in all the affairs of his living.

Yet the very teacher who strongly urges such views may *not* apply them to such a closely related field as that of *algebra*. We were to have learned through geometry to eschew all but logical deduction through-

out the whole course of our life, in all situations. Instead, we have hardly turned our backs on the geometry class until we face —the very next hour—a class in algebra. Do we then *transfer* this logical-reasoning training to that class? Why—*not at all!* There are no axioms this hour! We simply decide—in this class—that things are true because they appear to be reasonable; and *logic* is forgotten! We multiply two negative numbers to obtain a positive number, giving at most some *illustration* of some isolated instance. What was to last a lifetime did not last one hour!

Does logical-reasoning training transfer to law? I do not know. But I *do* know that years and years of repetition of logical deduction in geometry classes does not seem to cause the *teacher* (let alone the pupil) to worry about logical deduction in classes in algebra. Shall we expect *more* of the student who sees geometry for only a single year? Shall we expect transfer to law when it is absent from algebra?

Perhaps you will point out that the two subjects—geometry and algebra—are different, and that the methods of geometry do not apply to algebra. Is then *law* more closely related? You are aware, I think, that careful systems of axioms *do exist* for algebra, and that the facts of algebra can be deduced logically from them in the same sense that the facts of geometry are deduced from the axioms of geometry. These situations are therefore far more *identical* than are the situations in geometry and in law, for there are no carefully prepared sets of axioms in law from which the facts can be deduced logically.

Let me descend, however, to a case of more distinct identity in geometry and in algebra. In geometry, we often discuss with minute care the so-called incommensurable cases: that, for example, two arcs may not have a common measure. Two segments of straight lines, for example, the heights of two rectangles, may be "incommensurable." Often we maintain that nobody can comprehend (say) areas of rectangles, who does not know how to deal with these cases.

Are there no "incommensurables" in algebra? Is the square root of two less "incommensurable" than is the diagonal of a unit square?

You observe, of course, that I have now reached *complete identity*, for the diagonal of the unit square is entirely identical with $\sqrt{2}$. Yet no one ever bothers to talk about the amazing character of $\sqrt{2}$ in a course in algebra! Why? Does the training in dealing with incommensurables in geometry "transfer," even in the mind of the *teacher?* . . . to the *identical* incommensurables in the algebra book? It does *not.* In fact, the authors of textbooks on geometry who have stressed most insistently the so-called "proofs" of the incommensurable propositions are the very ones who have most completely ignored the same considerations in their own textbooks on algebra! Finally, lest anyone suppose that I am arguing for the introduction of the same fallacious proofs into books on algebra, let me remark that I have joined publicly with those who have recommended their elimination from courses in geometry: notably the Report of the Committee on the Reorganization of Secondary-School Mathematics. My purpose here has been to demonstrate that insistence upon these proofs in the texts on geometry has not *transferred,* either in the minds of the teachers nor of some of the most emphatic *authors* to the identical problem in their own courses in algebra.

Shall I mention other instances of failure of transfer? May I mention at least a few which happen to have occurred to me? In algebras, it is common to factor $y^n - x^n$ into $y - x$ and $y^{n-1} + y^{n-2}x + \cdots + x^{n-1}$. In the same book (this time not even in a different book) we teach that the sum of a geometric progression is

$$1 + x + x^2 + \cdots + x^{n-1} = (1 - x^n) \div (1 - x).$$

May I ask how many of you have noticed that these formulas are identical? They *are* identical, if only we set $y = 1$. Shall we

expect, without fail, that students will "transfer" these trainings to the study of such subjects as the study of compound interest on money? It is true that the formulas for compound interest are actually geometric progressions. Is that fact, that is, that the subjects concerned contain "identical elements" sufficient to ensure "some degree of transfer"? Without specific guidance? If it is, then each of you should have made the transfer from the factoring formula to the geometric progressions, and you have indicated by your nods of your heads to my question that you did *not* make that transfer without guidance, that is, until I had pointed it out to you just now.

The formula $(x+a)(y+b)$ is taught under factoring in algebra. You—teachers—often consider at a different place in your teaching how many figures should be kept when you (or your students) are multiplying together two numbers that are subject to error: thus if a measured reactangle has sides 5.8 ft. by 7.4 ft., the area is 42.92 sq. ft. You are accustomed to tell students to throw away the last "2". Has it occurred to you to use the formula stated just above? You know that such a measurement as 5.8 ft. indicates that the last figure given is accurate to within "5" in the next place of decimals. The numerical problem stated above is therefore really $(5.8\pm0.05)(7.4\pm0.05)$, possibly; and the possible error is therefore (by the factoring formula) perhaps as great as 0.66; why announce the result, then, as 42.9? The digit "9" is surely definitely very uncertain. However, the real point is that you may not have "transferred" the oft-repeated formula for $(x+a)(y+b)$ even to this easy business of the multiplication of numbers.

4. *Teaching for Transfer of Training.* It seems to me that we cannot reasonably expect *more* of the students than we find is true of ourselves. We cannot hope that *they* will think of applying $(x+a)(y+b)$ to find out how many figures to keep in a multiplication unless all *teachers* have seen the same opportunity for "transfer." I mean, of course, without having somebody explain the point: the obvious conclusion that I draw is that we *should* explain not only this point, but many, many, similar opportunities for "transfer." We should, in fine, *teach* for transfer of training, by many, many instances of this specific sort which show valid opportunities for transfer.

We cannot hope that the *student* will apply (or will even see the possibility of applying) geometric progression (or, what is the same thing, the factoring of $1-r^n$) to the problems of *compound interest* if the *teachers* have *not* transferred even from factoring to geometric progression. We can not hope that the student will transfer the habit of deductive reasoning to his whole life if the *teachers* have not carried it from one class into the next one. Guidance, actual teach!ng, is required to assure the simplest possible carry-over of this type.

It appears to me, therefore, that we shall get very little such "transfer" unless we make it a part of our business to *teach* it: every day and at every opportunity, in large matters, and in small numerical instances.

Is this "out of our field"? Is it "leaving the serious part of the subject"? Shall we take the time away from "more important things" if we show the students in no hazy way how to transfer what they are learning to other matters? I think it is a *part of our business* to do these very things. I believe that if we do not do them, the effectiveness of our training is lost to a very surprising degree—to an extent not commonly recognized. In fact, I think it is far more vital for us to teach the student how to transfer his training to different fields than it is to go through all details of all logical proofs.

Thus, I would sacrifice the proof that $(-a)(-b)=ab$, without much compunction, in order to gain time to bring out some of the possibilities of transfer that I have mentioned. The proofs given are usu-

ally bad, anyway. But I would like to have the student shown that 5.8×7.4 really means a possibility of $(5.8 \pm 0.05)(7.4 \pm 0.05)$, and that the result may therefore be anywhere between 42.26 and 43.58; in doing so he will effectively use the fact that $(-a)(-b) = ab$; and he may then really begin to appreciate that rule.

I would not hesitate to sacrifice the (for the most part fallacious) "proofs" of the so-called incommensurable cases in geometry in order to get time to show, for example, the "transfer" from the triangle propositions to the question of the rigidity of a bridge-framework, or to the proper bracing of a shelf, or a table-leg.

I would be willing to sacrifice some of the intricate proofs about fractional exponents in order to show him the connection with logarithms, and how they work.

In this brief hour, I cannot present to you every such instance. You must yourselves attempt to discover them. Such discovery is *not* easy, though it may appear to be so after the possibility of transfer is pointed out. The reason that it is not easy, is precisely because it is a case of that transfer of training, which does appear to be so strangely hard. Nevertheless, you will discover many, many such cases if you will appreciate that *it is your business* to to do so, and if you will bend your will whole-heartedly to the discovery of them and to the transmission of them to your students.

In general, *not* for their so-called "practical values," but distinctly because the transfer of training in mathematics is far from easy, I would like, at every point in mathematics, to show the student, not vaguely, but in definite detail, just how what he has learned can be transferred to other fields, to life, to his own life. Such is, in my opinion, our maximum task in the teaching of every branch of mathematics.

Annual Meeting of the National Council of Teachers of Mathematics!
Place—Palmer House, Chicago, Illinois
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See pages 94 and 95 of this issue of "The Mathematics Teacher" for the complete program.

Let's Face the Facts

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FOR a generation there has been considerable ferment with respect to the place and content of mathematics in the high school curriculum. Two central issues have been prominent: (1) Does mathematics as now taught constitute a more suitable content for the education of the great mass of high school pupils than other subject matter which might be substituted in its place? (2) Should the content of high school mathematics be thoroughly re-organized?

It is not the purpose of this paper to discuss the merits of any position which may be taken with respect to the first issue, but merely to point out that it is an issue—so much an issue that were it not for college entrance requirements, it is quite likely that not one pupil in four above the eighth grade would include mathematics in his program.

With respect to the second question; namely, "Should the content of high school mathematics be re-organized," it would seem that it is not only a question of "should" but also one of "must." It is to that problem that the major portion of this discussion will be directed.

Let us begin by reviewing certain statistical facts. The estimated enrollments in public secondary schools have increased as follows:

1900	519,000
1910	915,000
1920	2,200,000
1930	4,400,000
1935	6,100,000

In 1900, 56.3% of the 519,000 pupils in public high schools were studying algebra. This means that practically all freshmen were studying algebra and that there were perhaps one in four or one in three of the other classes also studying second year algebra. In 1935, instead of 56.3%, there are slightly more than 25%, or one in four*

In 1900, 27.4% of all high school pupils were studying geometry. This means that practically all students studied geometry in either the junior or senior year. In 1935 less than 15% of the pupils in high school are studying either plane or solid geometry.

While the *percentage* of high school pupils studying algebra and geometry has decreased, the *number* has increased very greatly. In 1900, only 291,000 pupils were enrolled in algebra, but in 1935 more than a million and a half pupils were so enrolled. Comparable figures for geometry reveal a like growth. This is attributable to the tendency to make secondary education universal. It is of great significance to those interested in the teaching of mathematics that high school enrollments have just about reached the saturation point.

It seems most probable that future enrollments in high school will probably be little, if any greater than they are this year. The depression with its effect upon employment and the relief measures of the state and federal governments have kept enrollments up. With the return of "prosperity" we may expect to see more youngsters leaving to work. A decreasing birth rate will also soon affect the situation. There are already fewer children in every elementary grade up through the fifth than were in those grades two years ago. In three more years the high school will experience the effects of this phenomenon. In the past we have been experiencing a rapidly increasing high school population and a slowly, though steadily, decreasing per cent taking high school mathematics. In the future we seem certain to experience *a stationary or decreasing high school population along with a decreasing per cent studying algebra and geometry.*

It will be useful also to note that the per cent of pupils taking mathematics after a year of algebra and one of geometry is but a fraction of those enrolled in the first two years. This may be safely attributed, for the very large part, to the fact that for entrance to most schools or colleges, two years of mathematics, no more and no less, has been required for entrance. Illustrative of this shrinkage are the following figures for the state of Minnesota:

Enrollments in high school mathematics, Minnesota (to the nearest 100)

First year Algebra	16,600
Plane Geometry	12,700
Second year Algebra	2,400
Solid Geometry	1,600
Trigonometry	500

It is useful but not pleasant to speculate upon what might happen if Minnesota colleges no longer required two years of high school mathematics. It is a most pertinent fact that colleges in this area have already begun to relax their requirements in mathematics. It is no longer necessary to present any units in mathematics for entrance to those divisions of the University of Wisconsin which are not directly dependent upon algebra and geometry.* Northwestern University has also moved in this direction. At the University of Minnesota, the University Senate last spring repealed its requirements of a specified pattern of high school credits for entrance, leaving it to each college to decide for its own students. The third largest division of the General College of the University, which receives students directly from high school, almost immediately announced that no mathematics would be required for entrance there. Most startling of all, the University of Iowa, the Iowa State College, and the Iowa State Teachers College rescinded last year the requirement of mathematics for entrance to those institutions. Mathematics enrollments in Iowa this fall are reported to have suffered heavy losses with even greater losses in view for next year. The University of Southern California, Leland Stanford University, and a score of smaller institutions no longer require high school mathematics for entrance. The question is being agitated in many states and will be decided soon in a number of them including Oregon and Nebraska.

It seems clear that this movement will spread fairly rapidly in the next decade. Within the past few years, from a number of carefully conducted studies, data have been reported with remarkable unanimity indicating that marks made in college seem to bear no relation to the pattern of subjects taken in high school. As illustrative of a score or more of similar studies, the results of a few will be stated briefly:

At Colorado State Teachers College, Gebhart[1] reported that students with less than two units of mathematics made just as high marks considering their intelligence as those with two units and no more, in fact, a little better.

At the same institution Stinnette[2] reports that those who lacked credits in mathematics or some other field required for entrance at the University of Colorado, University of Denver, or Colorado State College, did just as well as those who could have entered one of those three institutions.

In the fall quarters of 1932 and 1933 about a hundred freshmen who did not have the required pattern of entrance credits were admitted to the General Col-

* The University of Wisconsin, however, does set up certain requirements in mathematics. They have "restricted" or "unrestricted" admission—the unrestricted type applying only to those who have included algebra and geometry in their preliminary training. See Langer, R. E., "The New Mathematics 'Requirement' at the University of Wisconsin." *The American Mathematical Monthly*, April, 1935, pp. 208–212.—Editor.

[1] Gebhart, G. L., *Relative Value of College Entrance Subjects*. M.A. Thesis, Colorado State Teachers College, December, 1923.

[2] Stinnette, Roy L., *An Evaluation of the Present College Entrance Requirements at Colorado State Teachers College*. M.A. Thesis, Colorado State Teachers College, August, 1930.

lege of the University of Minnesota for experimental purposes. Most of these students did not have as much as two high school units in mathematics. With respect to scholastic record this was of no material or significant difference in the achievement of the experimental group and a comparable group of students in the general college who had presented the required number of units.[3]

In studies conducted by the author at the University of Oregon[4] and the University of Minnesota, it was shown that there was no correlation between the number of units taken in any given field and subsequent scholastic success in college.[5] Similar results have been reported by Sorenson,[6] Brammel,[7] Yates,[8] Bolenbaugh and Proctor,[9] Michaelson,[10] and Nelson.[11]

The practice of selecting of students for entrance to college on the basis of the pattern of subjects taken in high school seems to be indefensible, and confronted with the data from these studies, it is hardly possible that the majority of institutions will continue the custom. Many of the institutions in which those responsible for selling entrance requirements would prefer for some reason or other to continue to specify certain units for entrance, will find that the pressure is too great to resist. There will be a strong tendency to meet the concessions made by other institutions with whom they must compete for students.

To be sure, the student who wishes to enter some branch of engineering will find it absolutely necessary for the time being at least, to present for entrance at least a year of algebra and a year of geometry. In addition, the student who enters college without at least a year of algebra will find it impossible to succeed in some courses and very difficult to succeed in certain others.

Nevertheless, certain conclusions are inescapable: (1) The number of colleges and universities requiring mathematics beyond arithmetic for entrance is decreasing and is likely to continue to decrease. (2) As a consequence, enrollment in high school algebra and geometry, unless reorganized in content so as to possess greater values for non-college preparatory purposes, will experience a very marked decrease.

There are two alternative courses: (1) to stand our ground and let the colleges do what they may and to trust to the general appreciation of mathematics and to a campaign of educating the public with respect to the values of algebra and geometry and (2) to begin immediately to reorganize the content and placement of high school mathematics so as to be able to maintain enrollments on the basis of the "practical" values of high school mathematics.

In my own mind there is little choice. I am certain that to follow the first course

[3] Kronenberg, Henry H., *The Validity of Curricular Requirements for Admission to the General College at the University of Minnesota*, Ph.D. Thesis, University of Minnesota, 1935.

[4] Douglass, Harl R., "The Relation of High School Preparation and Certain Other Factors to Academic Success at the University of Oregon." *University of Oregon Publications*, Vol. III, No. 1, September, 1931.

[5] Douglass, Harl R., *Relationship Between College Marks and Amount of Credit in Different High School Subjects or Combinations of Subjects.* Board of Admissions, University of Minnesota, 1935. Mimeographed.

[6] Sorenson, Herbert, "High School Subjects as Conditioners of College Success." *Journal of Educational Research*, XIX: 182–192, April, 1929.

[7] Brammel, Paris R., *A Study of Entrance Requirements at the University of Washington.* Ph.D. Thesis, University of Washington, 1930.

[8] Yates, J. A., "The Type of High School Curriculum Which Gives the Best Preparation for College." *Bulletin of the Bureau of School Service*, University of Kentucky, Vol. II, No. 1, September, 1929.

[9] Bolenbaugh, Lester and Proctor, Wm. M., "Relation of Subjects Taken in High School to Success in College." *Journal of Educational Research* XV: 87–92, February, 1927.

[10] Michaelson, Jessie H. and Douglass, Harl R., "The Relation of High School Mathematics to College Marks and other Factors Pertaining to College Marks and Mathematics." *School Review*, XLIV: 615–19, October, 1936.

[11] Nelson, N. J., "A Study in the Value of Entrance Requirements of Iowa State Teachers College." *School and Society*, 37: 262–65, February 25, 1933.

is sure to result in the disappearance of geometry and perhaps algebra from the offerings of thousands of small high schools because the number electing those subjects will be too small to warrant offering sections in them.[12] In the schools continuing to offer algebra and geometry enrollments will shrink materially and steadily every year for a number of years.

We should not deceive ourselves about the matter. Parents give us much lip service with respect to the values of mathematics, but once college entrance requirements are removed, parental insistence upon those subjects will hardly be comparable to what teachers of mathematics might reasonably expect in the light of the statements of parents of their belief in the values of mathematics.

In the years past we have been guilty of some pretty loose thinking about the reasons why pupils should study algebra and geometry. Much ink and paper has been consumed by articles written on the educational values of these subjects—so much as to give the impression of whistling in the dark—enough to suggest that mathematics teachers themselves had begun to have doubts and that they felt it necessary to keep each other's courage up.

It is frequently posited, for example, as a reason for teaching mathematics that we owe our glorious mechanical civilization and our knowledge of science, particularly astronomy and physics, to advanced mathematics. That these contributions were made by a relatively small number of well trained people seems not to be remembered. With few exceptions those making the contributions on which we congratulate ourselves, received their secondary education prior to 1910. In 1910, there were in high schools less than one sixth the number now attending high schools and less than half of these graduated. In other words, it would, on the basis of this claim, not be necessary to teach high school mathematics to more than about 10% of those in high school today.

It will not be effective to attempt to defend our present courses on the basis of "transfer of training." Granting that there is considerable transfer, even as much as claimed in the ridiculous and juvenile statements sometimes made, it cannot be successfully maintained that there is distinctly more transfer than other subjects possessing, in addition, more directly practical values of the mass of high school pupils. Even were the experimental results of Thorndike[13] on the matter of the relative discipline value of the study of mathematics as compared to other subjects to be reversed by future investigation, it is not reasonable to expect that those considerations, however valid, will be appreciated by parents and pupils sufficiently to maintain enrollments.

It seems practically certain that unless the content of high school mathematics is immediately re-organized before the supporting braces of college entrance requirements are withdrawn, we will experience a deflation very similar to that which has occurred in Latin. The Latin people were similarly warned, but they chose not to listen though it is quite possible that, unlike mathematics, Latin possessed little possibilities in the way of revision better to fit the needs of the high school pupils.

At any rate, the Latinists chose to defend the *status quo* and to belittle their critics. Beginning about 1920, they became really alarmed. Latin students were not increasing in the proportion as were students in other fields. Since that time, frantic efforts have been made to "reorganize" and to sugar coat their subject. The new textbooks are markedly different from those which prevailed for a hundred years previously. But it was too late, the

[12] The median enrollment in high schools in the U. S. is very close to 105 and the median number of pupils in the ninth and the tenth grades approximately 38 and 28 respectively.

[13] Thorndike, E. L., in *Journal of Educational Psychology*, 15: 1–22; 83–98; reports no significant differences between various high school subjects in their contribution to general mental abilities.

swing was too well under way to be stopped. In 1900 more than sixty per cent of the students in secondary schools were studying Latin. In 1935 the figure is less than twenty per cent.

Even were we willing to stand by and see enrollments in mathematics experience a similar nose-dive, there are other reasons why we should attack the problem of re-organizing the curriculum. Mathematics has come to play a much larger part in the life of people today than formerly—in agriculture, in investments, in purchasing, in problems of diet, in shopwork, in borrowing money from "loan" companies, in "finance" plans and installment buying, in home planning. In addition, there is among the six and a half million high school students today a far greater proportion of boys and girls who will be clerks, mechanics, factory workers, housewives, farmers, workers in what we class the lower vocational classifications as compared to professional and high grade business executive categories. These people—constituting probably seventy-five or eighty per cent of the freshmen and sophomore classes in high school, as well as those who will go to college, require the extension of arithmetic, geometric constructions, intuitive geometry and formula-equation algebra rather than college preparatory algebra and demonstrative geometry.

There is a very definite and pressing need for instruction in the applications of arithmetic and simple algebra to life situations which require quantitative thinking, much beyond that now offered in arithmetic as taught in grades seven and eight. Considerable of the material now taught in grade eight should be transferred to grade nine and taught to pupils who are a year more mature and understand better the business, industrial and scientific background of the mathematical computation.

This is not the place to present a detailed outline of the re-organized course for grades 7, 8, and 9. I have in my files several such outlines. In the Minneapolis schools, as well as a few other centers, similar tentative outlines have been developed. They are of two types: (1) optional course in ninth grade mathematics and (2) a required core for grades 7, 8 and 9.

In the "optional" plan, instruction in grades 7 and 8 is the same for all students and rather similar to what is usually taught in these grades. In the ninth grade two courses in mathematics are offered: (1) algebra and (2) the "optional" non-college preparatory or "consumers" mathematics consisting largely of arithmetic and the simplest and most practical phases of algebra.

In the "required core" plan, which seems to me to possess very decided advantages, but one ninth grade course is offered. The courses in seventh and eighth grade mathematics are re-organized, admitting new and additional material relating to applications of arithmetic to home, shop, farm, store, health, transportation, etc., and also involving a shift of a few topics from the seventh to the eighth grade and several from the eighth to the ninth grade. In the ninth grade appears some material shifted over from the eighth grade, with more complicated and life-like problem material, some new topics not now taught in arithmetic, much use of the formula and simple equation, simple statistical method as applied to situations frequently met, and brief exploratory sections in algebra and geometry.

This three year sequence should be required of all students. In the tenth grade following such a sequence should be given an elective thorough course in algebra, completing "first year" algebra and also including intermediate algebra. With the start already made in algebra in the required junior high school sequence, and the more select and more mature tenth grades as compared to ninth graders now studying algebra, this should be easily possible. Similarly in the eleventh grade, plane and solid geometry might well be

completed in one year, leaving the twelfth grade for trigonometry and advanced algebra.

There are certain distinct advantages in such a plan as outlined above. Chief of these are:

1. A much more effective course in junior high school mathematics for all students, college preparatory and non-college preparatory.

2. The greater mastery of algebra certain to result from the favorable condition afforded in the form of a select mature group in the tenth grade.

3. The greater mastery of algebra resulting from the additional half year of intermediate (elementary) algebra.

4. The lessened time elapsing between the completion of algebra and college entrance.

5. The postponement of algebra as such to the senior high school where it will in many instances be taught by better prepared teachers than those in the junior high school.

6. A favorable effect upon enrollments.

The last named advantage requires some detail. Where the required course plan is employed enrollments in the ninth grade are certain to be greater by from 25% to 50% than enrollments in algebra now in that grade and probably two to five times as great as enrollments after algebra is no longer required for entrance to arts colleges. In the tenth grade the numbers enrolled in algebra would surely be greater than would be enrolled for geometry should algebra be continued in the ninth grade and geometry in the tenth. In the eleventh grade there should be many more enrolled in geometry than would otherwise be enrolled in whatever courses as may be offered beyond geometry in such schools as would offer any. In the twelfth grade it is difficult to speculate with any assurance.

It may also be of some interest that the large majority of professors of engineering, if the score or so with whom I have discussed the matter may be taken as a reliable sample, prefer distinctly that all their students have had a year of combined plane and solid geometry than to have some students attempting to make up solid geometry during their college freshman year, and also that they would prefer an additional half year of algebra even if it means spending no more than a year on plane and solid geometry combined.

It is clear to those who have attempted to remain in close touch with trends in thinking and practice in secondary education that the curriculum is just entering upon an era to be characterized by an unusual amount of re-organization. Readjustments to the new universal student body are long overdue. A re-awakening to the need for civic, economic and leisure education is accelerating the revolution. College entrance requirements are definitely in the period of relaxation and colleges are turning to measures of ability and forsaking patterns of exposure as a means of selecting their students.

It does not seem wise for teachers of mathematics to shut their eyes and ears to these developments and their implications. It seems much more intelligent to attack the problem open-mindedly, with the objective attitude to the development of which the study of mathematics and science should contribute, and with rational rather than emotional methods. At any rate, those of us who think we see the signs of the time would be derelict in our duty if we continued to tell teachers of mathematics what they would like to hear to the exclusion of what they ought to know.

One is reminded of Jeremiah of biblical history who had a habit which the philosophers praised but his people disliked. He insisted upon telling the truth to those whom he thought it needed to be told. He was despised, reviled and thrown into prison.

But he was right. He warned his people not to trust to some vague and remote authority. He ridiculed them as being blind to developments, crying peace when

there was no peace. His unwelcome prophecies came true. Jerusalem was captured, the temple of Solomon destroyed and Judah driven into Babylonian bondage. They knew then that the sour old prophet had been right. But it was too late, Jerusalem was gone and so was Jeremiah, the old joy-killer.

Some of the ideas presented above have been presented to groups of mathematics teachers to my personal knowledge for ten years or more and they have often, particularly in the earlier years, been received with poorly concealed displeasure, often followed by a good old rabble-rousing pep talk by an optimist or opportunist. But already the walls of Jerusalem are crumbling. Some are rushing to build more substantial defenses, other stand by apparently either indifferent or panic stricken, others are still repeating time honored phrases of loyalty to the *status quo* long since bereft of meaning and validity, but which serve to shut out unpleasant thoughts. There is still time if we start now.

THIS article has been written from the point of view of the mathematics teacher as well as from that of one engaged in general secondary education. It has been written by one who has long been interested in and associated with the teaching of high school mathematics; whose undergraduate teaching major was mathematics and who observed almost daily the teaching of mathematics for an additional ten years and who taught high school mathematics for six years; who devised three series of standard tests for algebra sold commercially for a number of years; who has twice reviewed and summarized for publication in the American Educational Research Association the research in the teaching of high school mathematics and is engaged in preparing a third report in that field; and who for ten years has been a member of the National Council of Teachers of Mathematics; who has met with and taken part many times in the programs of groups and clubs of mathematics teachers in Oregon, Minnesota, Iowa, Nebraska and Illinois. These things are mentioned lest some may question the sincere appreciation of secondary school mathematics and keen interest and unbroken contacts of the author for more than twenty years with the problems of high school mathematics.

Because of the blunt and often superficial and unintelligent nature of many criticisms of mathematics in the high school curriculum made by outside critics there has developed a tendency on the part of many teachers of mathematics to receive criticisms with more antagonism than understanding. Therein lies a danger. Unless we are willing to sell mathematics down the river in order that we can excuse ourselves from the intellectual efforts of revising the content of high school courses we must examine criticisms carefully and objectively. We must survey recent and current developments and trends thoroughly and be guided by what we see. It seems that the day is over when an united front of opposition rallied around the defense of the *status quo* will suffice. Newer and greater problems perplex us than ever before. We must face the facts and make appropriate adjustments.

. . . MANY will recall samples of Barrett Wendell's wit, among others an unusually good pun. He was constructively on a leave of absence and appeared at a department meeting. A colleague remarked that he should not be present, that he was *non est;* whereupon Wendell flashed back: "A non est man's the noblest work of God."—ARTHUR LYON CROSS, in *The Michigan Alumnus.*

An Historical Excursion

By A. J. Cook

University of Alberta, Edmonton, Canada

My youthful education was rather sadly neglected; no doubt, chiefly the result of my own obtuseness. At any rate I seemed to have moved for a long period within the penumbra of mathematics. Some considerable time elapsed, in spite of my interest in the subject, before it became apparent to me that mathematics was cultural in its quality, that the sciences were, or at least should be, humanities, and that the feud that seemed so often to rage over so-called literary and so-called scientific matters was a battle of prejudice, at least as much as a battle of wits.

Perhaps I became first conscious of the sheer wealth of the mathematical traditions in listening to some lectures of Professor W. F. Osgood on *The Theory of Functions of a Complex Variable.* His comments on the people behind the theory had a vital quality, which usually served to enhance one's interest in the theory itself.

For various reasons chiefly, though I do not think wholly, personal, the thing I had begun to see was not fostered during a rather protracted and interrupted period of graduate study. I hungered to read at least a few of the masters, but there never seemed to be time enough left over from lecture courses. Nor did the subsequent roll and pitch of teaching encourage the thought, though I never lost the longing to visit some of the historical sites of mathematics.

An opportunity to do this very thing presented itself recently. Three or four months ago, when cogitating upon the connections between projective and metric geometry, it occurred to me that I might try to ferret out what the Greeks knew about projective matters.

I spent a morning browsing around the library, dipping into books on Greek geometry and architecture, on geometrical optics, mathematical history, modern geometry, etc. The book that came home with me was Chasles, *Aperçu Historique.*[1] I read the historical part of the book through in the next few days and enjoyed, in the phrase of R. C. Archibald, a "thrilling intellectual experience." An *Aperçu* indeed! An aeroplane chartered at zero cost; a continent surveyed, known previously to me only by the place names.

It seemed clear that the next step was detailed appreciation, and so notes on the first period were begun.[2]

It was not long before Heath's monumental history of Greek mathematics was beside me, enabling me to expand and to amend the brief observations of Chasles.[3]

Here follows a list of things which were news to me:

(a) The origin and probable first constructions of the conic sections.
(b) The extent of Euclid's writings on conics.
(c) The proofs of Apollonius of the ordinate properties of the conics.
(d) The Pythagorean origin of the words *parabole, ellipsis, hyperbole,* and their use by Apollonius.
(e) The setting of the three classical problems of Greek geometry, within the Greek story.
(f) The origin of the method of exhaustion; the detail of its application by Archimedes.
(g) The nature and mystery of the *Porisms* of Euclid.

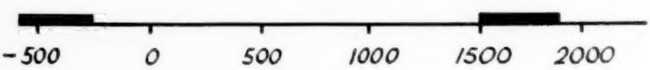
[1] M. Chasles, *Aperçu Historique sur l'origine et le développement des méthodes en géométrie* (Paris, Gauthier-Villars, 1889).

[2] Chasles divides the history of geometry up to his day into five periods of which the first extends from Thales (−659– −548) to Proclus (412–485).

[3] T. L. Heath, *History of Greek Mathematics,* 2 vols. (Oxford Press).

Now it so happened that at about this time summer school at the university found me with a class of thirty-five in analytical geometry and the calculus. What I had freshly learned seemed to me to have possibilities for these students—many of them rural high school teachers. Yet I resolved to be sparing in my enthusiasm, for our teaching time totalled forty-five hours, and students had to be prepared in that period for their winter home study.

I began the course with a diagram, shown below:



Time Scale

The diagram was intended to be a rough picture of the extent of the creative periods in mathematics (and the sciences). Then followed a very brief outline of the origins of Greek geometry, of the growth of algebraic symbolism, and their explosive union in the work of Descartes and Newton.

We then proceeded on our analytic way, but not before the definitions of Viete of the Analytic and Synthetic methods were written on the board in the words of Chasles (*Aperçu*, p. 5).[4]

The translation was made there and then by one of the students; the meaning came later.

From the start of the course books were brought to the class-room for brief exhibition, including the Rhind Papyrus, the *Elements* of Euclid, the conic sections of Apollonius, the geometry of Descartes, and Newton's *Principia*.

Four historical talks were planned, to be given at appropriate times. The first two of these were brief reviews of two books, one on mathematics in ancient Greece, and the other on the history of European mathematics.[5] Both of these talks were given by students who were frank to admit the many things they had not yet appreciated. Thus the first speaker confessed he did not know what it meant to square a circle, nor did he see why an angle could not be trisected. What struck me most forcibly about these talks was the possibility of "research" which each speaker set before us. After the second talk for instance in which the speaker made a passing reference to the invention of logarithms, one man sought further information on the matter. When brief mention was made of arithmetic and geometric series, the enquirer remarked that he began—as teacher—to sense for the first time the significance of series in mathematics.

The third of the talks was given after the analytic approach to the conics had been completed, and comprised: (1) the origin of the conics, (2) the development of the ordinate properties by the method of Apollonius.

The responses were interesting. One man said: "Those proofs are simple enough for a high school boy to understand, and I don't see why he shouldn't have them;—that was a revelation!" Another said: "A couple of us were discussing the development you gave us. What struck us was the fact that although we had studied a little in Greek History and Philosophy, these things were never mentioned. We wondered why we had not heard of such genius before."

The fourth talk was planned to take

[4] "Il est en mathématiques une méthode pour la recherche de la vérité, que Platon passe pour avoir inventée, que Théon a nommée analyse et qu'il a définie ainsi: *Regarder la chose cherchée comme si elle était donnée, et marcher, de conséquences en conséquences, jusqu'a ce que l'on reconnaisse comme vraie la chose cherchée. Au contraire, la synthese se definit: Partir d'un chose donnée, pour arriver, de conséquences en conséquences, a trouver une chose cherchée.*"

[5] J. L. Heiberg, *Science and Mathematics in Classical Antiquity*, (Oxford Press). J. W. N. Sullivan, *The History of Mathematics in Europe*, (Oxford Press).

place after the introduction of integration as a process of summation, to illustrate the Method of Exhaustion as applied by Archimedes. As it happened, there was time only to indicate briefly the scope and nature of the work of Archimedes; the detail of the theorems of Archimedes had to be omitted.

My experience with this class has satisfied me that the four hours out of a possible forty-five hours were well spent on background materials, which were not "in the text-book." Moreover, I am resolved to practice book exhibits as a regular part of class-room routine.[6]

Perhaps enough has been written to indicate the stimulus and interest which accrued from a rather brief excursion into the history of elementary mathematics.

[6] Incidentally, two-thirds of the class placed orders for R. C. Archibald's *Outline of the History of Mathematics* (Mathematical Association of America), after having read a description of it in the *Mathematical Monthly* (June-July, 1936, p. 383).

What, one may ask, of the original query? The first part of the answer lies in (g) of the earlier list. Euclid's three books of *Porisms* were probably "propositions belonging to the modern theory of transversals and to projective geometry" (Heath, vol. I, p. 436). This inference is due to Chasles, and is based on work that is historically important "because it was in the course of his researches on this subject that he (Chasles) was led to the idea of anharmonic ratios" (Heath, vol. I, p. 436).

The rest of the answer lies directly ahead. The exploration of the first period is still incomplete. The *Collection* of Pappus remains to be examined, even if we already know something of its significance. These things, however, are but beginnings. The remaining four-fifths of the landscape, as depicted by Chasles, beckons the traveller. A wealth of fascinating detail awaits disclosure, for Chasles' sketches of the work of Desargues, Pascal, Fermat, and the rest are enticement and invitation.

Mathematics and the Healing of Wounds

CERTAIN phenomena express a general modification of the organism. For example, the rate of healing of a superficial wound varies in function of the age of the patient. It is well known that the progress of cicatrization can be calculated with two equations set up by Lecomte du Noüy. The first of these equations gives a coefficient called index of cicatrization, which depends on the surface and the age of the wound. By introducing this index in a second equation, one may, from two measurements of the wound taken at an interval of several days, predict the future progress of repair. The smaller the wound and the younger the man, the greater the index. With the help of this index, Lecomte du Noüy has discovered a constant that expresses the regenerative activity characteristic of a given age. This constant is equal to the product of the index by the square root of the surface of the wound. The curve of its variations shows that a twenty-year-old patient heals twice as quickly as a forty-year-old one. Through these equations, the physiological age of a man can be deduced from the rate of healing of a wound. From ten to about forty-five years, the information thus obtained is very definite. But later, the variations of the index of cicatrization are so small that they lose all significance.—ALEXIS CARROL in "Man the Unknown," pages 167–8.

Is Mathematics Teaching on the Secondary Level on the Spot?

By W. J. Klopp
Supervisor of Secondary Schools, Long Beach, California

In beginning the 1935 and 1936 series of Educational Symposia on the secondary level, for the various departments, it occurred to me that it would be quite in harmony with trends in progressive education, to have each department take for its major theses for discussion, certain definite outcomes which we have a right to expect as a result of experiences provided by the secondary schools.

I feel now that I am not asking too much of departments on the secondary level to react to the following theses in the light of their own subject matter, wherein the child finds himself experiencing. I feel especially justified in making these requests on the grounds that education is conceived of as a gradual, continuous and unifying process, meaning, of course, that education begins at births and ends with death, and that during the conscious hours of any individual, the educational process is in operation. Since we become educated through learning, and since learning is experiencing, and experiencing results in the modification of behavior, there appears to be no other alternative for the teacher of any subject than that of placing greater emphasis upon the outcomes expressed in terms of understanding, appreciation, interest, habits, and skills. We might add to these outcomes, however, the postulate that while the child is experiencing within any medium, it should be possible for him to make conscious evaluations of his own experiences. To this end, then, we have framed the first thesis to read as follows:

How May a Child Gain Certain Understandings, Appreciations, Skills and Interests Through the Medium of Mathematics by Means of Which He May Evaluate His Own Experiences?

Dr. E. R. Hedrick, Professor of Mathematics of the University of California at Los Angeles, presented this theme to about sixty teachers in the junior and senior high schools and the junior college. Dr. Hedrick pointed out to the group his startling observation that mathematics teachers were practically teaching processes and the application of principles without being concerned with the meaning of either experience. For example, he asked the group whether in the teaching of the linear equation it ever occurred to them that there were very specific situations in life which would be natural to the average child, and which might be used to illustrate the place and the true function of the linear equation in life. He admitted that it was a difficult thing for the average instructor in mathematics to think of enough life situations which might be revealed to a class engaged in the study of the linear equation or the simple quadratic, or factoring certain mathematical expressions, so that all of these experiences might have more significant meaning to the average child.

I think the group agreed, without question, that the present method of instruction in the senior high school, as well as the junior college, was quite adequate to pass examinations and receive grades for admission to a senior college or university, but that in both cases the child learned very little about the meaning and true function of mathematics as related to life.

Dr. Hedrick made a very definite plea for a richer list of illustrations of life situations, to give fundamental mathematical principles and concepts more significant meanings to the child, and he made a further plea to all mathematics teachers to endeavor to give the child a richer background of mathematical concepts and mathematical experiences, and put aside the unwarranted notion that the university requires that all teachers pre-

pare all students for entrance in these higher institutions of learning.

The second symposium theme was presented by Dr. L. D. Ames, Professor of Mathematics, University of Southern California, and stated as follows:

How May Good Thinking Be Developed in the Child Through the Study of Mathematics? meaning, of course, good thinking from the standpoint of scientific thinking, accurate and valid thinking, as well as thinking on a higher ethical plane. Dr. Ames bemoaned the fact that in our mathematics instruction on the secondary level, we departed from the old Euclidian idea and divested all mathematical teaching of all form of logic until the subject matter has become so diluted as to be absolutely without value. Dr. Ames stressed the fact that too much of the mathematics instruction in the secondary schools consisted in the solving of problems to meet the requirements of assignments by the instructor for the ability to pass examinations which followed the series of assignments. I think he expressed his idea on the theme quite clearly by stating that the only contribution the mathematics teachers were making to the child's equipment was through developing ability to follow directions, and that little thinking was required in the present methods of instruction. However, he pointed out the possibility of developing not only the good, valid, scientific thinking through the medium of mathematics, but also ethical thinking because of the infinite relationship of elements in all types of life situations which have mathematical significance. He, too, made a very dynamic plea for a richer background of illustration by the teachers of mathematics, to be offered to the students on the secondary level.

The third thesis presented for the evening, was presented by Dr. Skarstedt of Whittier College, and was entitled:

How Does the Study of Mathematics Contribute to the Growth of Children Toward Emotional and Social Maturity?

Dr. Skarstedt made it very clear to the group that learning and experiencing in the field of mathematics embodied all the essential elements of cultural growth, both in the realm of the emotional and the social, and that the child who became conscious of a power to solve problems and ascertain their meaning, was equally conscious of growth toward maturity.

It seemed to me that the entire symposium was unique and exceedingly stimulating because it evoked a different type of reaction from the average teacher of mathematics than was true in previous conferences where we discussed method and content. I am absolutely conscious, after this experience, which followed the experiences of four other departments, discussing the same themes, that this is an opportune time to present to teachers on the secondary level these outcomes for consideration in the light of their own subject matter, in order that the teachers of today might focus their attention a little bit more clearly and more resolutely upon the child, and observe the child's reaction to his experiences in any specific field. We may then discover just how he evaluates those experiences for himself.

I certainly would be the last one to say that mathematics is put on the spot by progressive education leaders, because I am conscious of the fact that mathematics is playing just as important a part in the life of a child's growth and development as any other single subject in the entire curriculum. However, I feel that every academic subject is on the spot insofar as making the desired contribution to the growth and development of the child is concerned. We must re-emphasize the value of outcomes so that with the goals clearly in mind we may guide the learning process more carefully and economically for the child toward the realization of these ideal goals.

It is my plea to all teachers of mathematics, that we give greater emphasis to meaningful experiences and less emphasis to the solution of problems. Problem solving is a valuable experience, but absolutely without merit if there is no meaning ascribed to the experience.

The Failure in Required Mathematics at Hunter College

By Laura Guggenbühl
Hunter College, New York, N. Y.

Introduction

During the Spring of 1934, plans for the revision of the curriculum at Hunter College were initiated. Questionnaires, asking for expressions of opinion concerning a "hypothetical curriculum" were distributed among members of the staff. There was a great deal of discussion with repeated references to the student who "simply could not do mathematics." It was surprising to see how many members of the staff believed that there was a person, of normal intelligence, who had satisfactorily completed necessary prerequisite courses, a person with normal background and training, willing to spend the required amount of time and thought in study and preparation, who could master every obstacle except mathematics. Some of the statements which were made seemed like a challenge to some of the members of the staff in the Mathematics Department. But when confronted with the questions,—"Why is the percentage of failures so high in mathematics" and "What picture does the failure in mathematics present,"—what could one answer? One could always say that Miss A failed because of lack of diligence; or that Miss B failed because she was inadequately prepared for the course; but one did not have sufficient data to venture from the particular to the general. Thus the following investigation concerning the failure in prescribed mathematics was undertaken.

The survey is based upon the records of students who failed Mathematics 1[1] in 1933. A complete record,[2] from the time of her entrance to college to June 1934, was obtained for every student who failed Mathematics 1 in January or June 1933. In January 1933, 212 students failed, and in June 1933, 107 students failed. However since some students failed in both January and June, the total 319 Fs were made by only 286 students. The following investigation is based upon the records of these 286 students.

The discussion is divided into three sections. In Section 1 there is a discussion of the influence of the preparatory school, the influence of the college teacher to whom a student is assigned, and the influence of the aptitudes of a student (as indicated by her choice of major subject) upon her chances of failing Mathematics 1. Section 2 is devoted to the study of special cases. In Section 3 one will find an analysis of the college records of the 286 failures.

Section 1

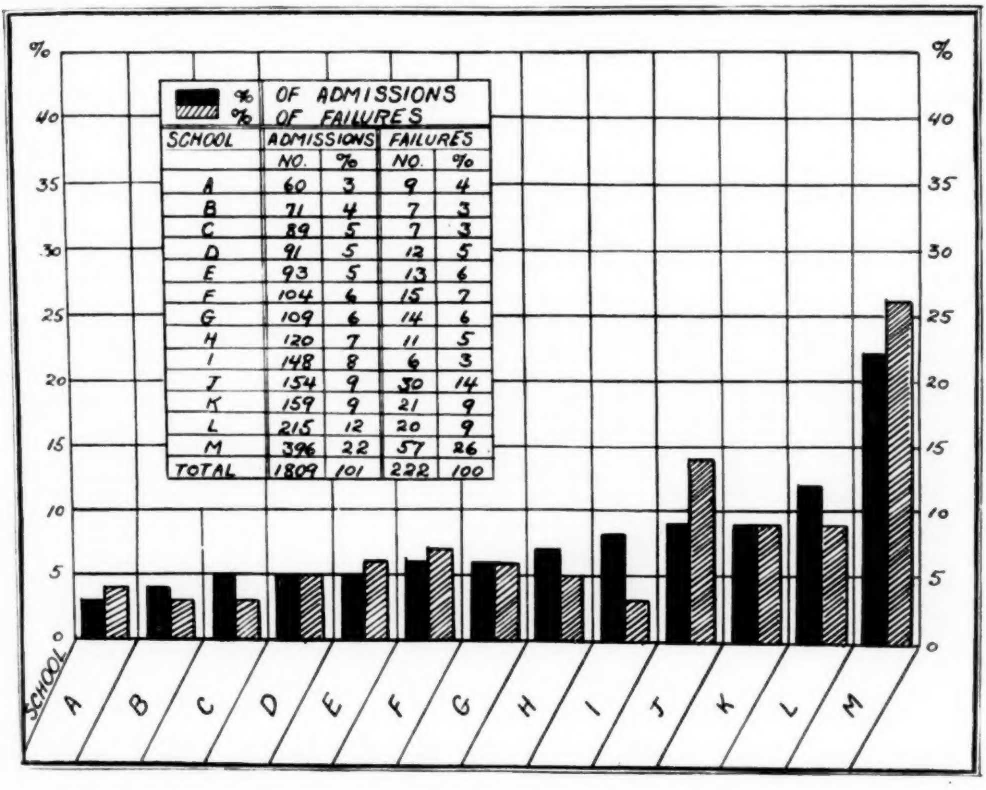
It is the purpose of this section to show that preparatory school, college instructor, and student aptitudes, may practically be disregarded in a discussion of the failures in prescribed mathematics. For this purpose three graphs are presented. In each, a comparison is made between a distribution of failures and a distribution of certain totals. A complete description accompanies each diagram, and comments are made on all three simultaneously at the end of the section.

During the college year, September 1932 to June 1933, 1809 students were admitted to Hunter College from various preparatory schools, most of which were

[1] Mathematics 1 is a forty-five hour one semester course which contains a unit of trigonometry, a unit of analytic geometry, and a unit of differential calculus. It is the course which most students take to satisfy the mathematics requirement at Hunter College.

[2] Throughout the discussion, grades in Choir and Orchestra, each a one half credit optional course in the Department of Music, have been omitted. The reason for this omission is that these grades were only A or F, F only because of excessive absence.

New York City high schools.[3] Private schools have been combined into a single group and treated as a single preparatory school. Every school for which the total number of admissions to Hunter College for this period was less than 50 has been placed in the miscellaneous group of schools, a group which is designated School M in the diagram in Graph 1 above. Individual high schools, for which admissions numbered more than 50, are designated A, B, C, and so forth. The reader will observe that instead of a total of 286 failures, the total 222 is used in calculating percentages of failures. In order to make the bases of comparison in the two distributions as nearly alike as possible, those of the 286 failures who had been admitted to Hunter College before September 1932, and those who had been transferred from the New York City Training Colleges for Teachers,[4] have been omitted here. The remaining 222 failures have been classified according to the schools A, B, C, . . . at which they were prepared for college.



SCHOOL	ADMISSIONS NO.	ADMISSIONS %	FAILURES NO.	FAILURES %
A	60	3	9	4
B	71	4	7	3
C	89	5	7	3
D	91	5	12	5
E	93	5	13	6
F	104	6	15	7
G	109	6	14	6
H	120	7	11	5
I	148	8	6	3
J	154	9	30	14
K	159	9	21	9
L	215	12	20	9
M	396	22	57	26
TOTAL	1809	101	222	100

GRAPH 1. Comparison Between Percentages of Failures and Percentages of Admissions from Individual Preparatory Schools.

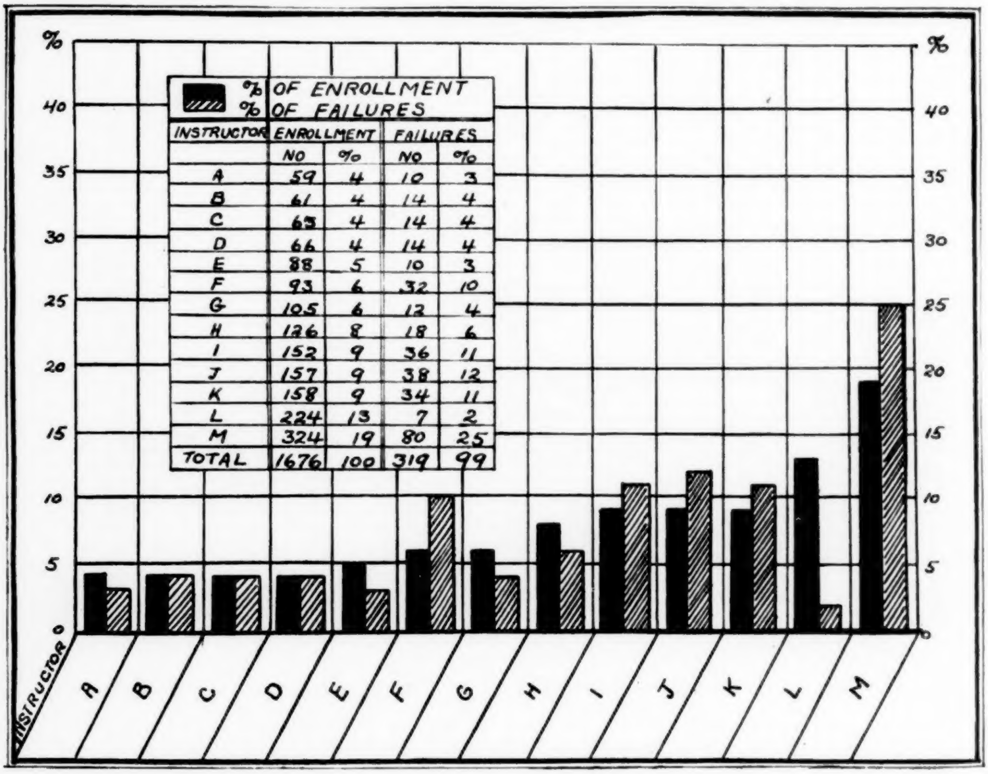
In January and June 1933, there was a total enrollment of 1676 students in Mathematics 1 distributed among 13 instructors, designated A, B, C, and so forth.

[3] There were also admitted to Hunter College during this year some students with advanced standing and some students from New York City Training Colleges for Teachers. Since these students had already had work of college grade before entering Hunter College, they have been omitted from calculations in the discussion concerning the influence of the preparatory school and do not represent any part of the above 1809 students.

[4] During the year 1932–1933 the New York City Training Colleges for Teachers were closed. The students enrolled at the Training Colleges were distributed among the other colleges maintained by the City of New York.

Since the smallest number of students enrolled with an individual instructor was 59, no miscellaneous group has been provided for this distribution. Here the reader will observe that the total 319, rather than the total 286, has been used in calculating the percentages of failures, due to each instructor. The reason is, of course, that the 286 students made a total of 319 Fs.

In calculating percentages of failures, the total used in 182. Of the original 286, 104 have been omitted for the following two reasons; 32 because they had been transferred from a training college and not required to choose a specific major, and 72 because they have changed their majors at least once. The remaining 182 were classified according to the grouped majors indicated in Graph 3.



INSTRUCTOR	ENROLLMENT NO	ENROLLMENT %	FAILURES NO	FAILURES %
A	59	4	10	3
B	61	4	14	4
C	63	4	14	4
D	66	4	14	4
E	88	5	10	3
F	93	6	32	10
G	105	6	12	4
H	126	8	18	6
I	152	9	36	11
J	157	9	38	12
K	158	9	34	11
L	224	13	7	2
M	324	19	80	25
TOTAL	1676	100	319	99

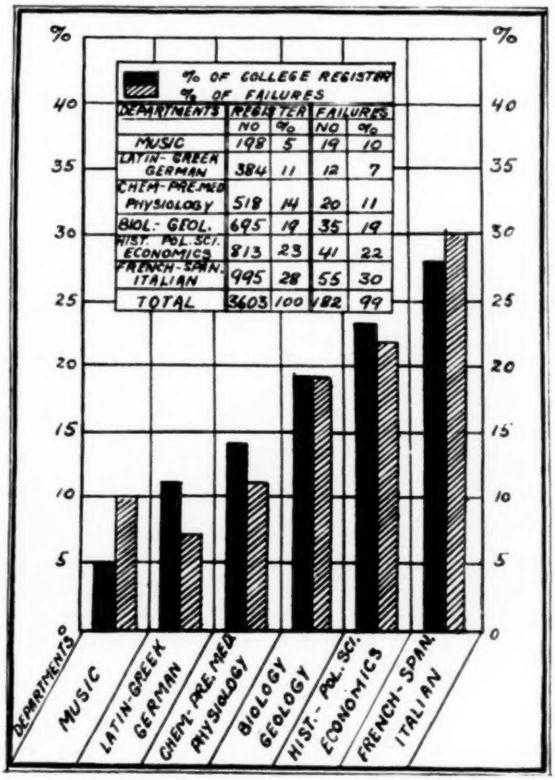
GRAPH 2. Comparison Between Percentages of Failures and Percentages of Total Student Enrollment for Individual Instructors.

In November 1932, the total student register at Hunter College included 3603 students distributed among the grouped majors indicated in Graph 3 shown below.[5]

The comparisons above are such as would naturally be made. For example, whereas the college instructor would be inclined to anticipate greater numbers of failures among students from one school rather than from another, the preparatory schools would be apt to ask for the

[5] There were also 35 majors in English, 11 majors in Art, and 664 majors in Mathematics. Since most of the failures in Mathematics 1 are freshmen and since no freshmen could be enrolled as majors in the Departments of English and Art at the time of this report, the 46 majors in these departments have been omitted here. The 664 majors in Mathematics have been omitted since students who signify their intention of selecting the Mathematics Major at entrance to college are not assigned to Mathematics 1.

name of the college instructor when searching for reasons for failure among students from their schools. Surely the most outstanding characteristic of the above graphs is the striking similarity between the two distributions in each diagram, for each school, for each instructor, and for each grouped department. In connection with the first graph, just one detail should be given by way of explanation. structor L, at the time covered by this report, was just beginning a teaching career under new and strange circumstances. Graph 3 is unusually satisfying. Whatever divergence from perfect accord is found in the two columns for each grouped department is a divergence in the direction which one would anticipate. The fact that there are relatively a few more failures in Mathematics 1 enrolled in the



Graph 3. Comparison Between Percentages of Failures and Percentages of Total Student Enrollment in Grouped Departments.

School I is one which selects its pupils at entrance on the basis of competitive examinations. Thus there is a very good reason for which School I should be an extreme in the comparison between percentages of failures and percentages of admissions from individual preparatory schools. In connection with Graph 2, the reader will probably wonder about Instructor L. Suffice it to say that Instructor L. Department of Music, than there are enrolled in the group of Departments, Chemistry, Pre-Medical Sciences, and Physiology, is insignificant. Once again, the striking characteristic is that the percentages of failures in the indicated grouped departments are so nearly the same as the percentages of students enrolled in these departments.

In summarizing, it may be said that a

girl's chances of passing Mathematics 1 depend but to a very small degree upon the preparatory school from which she comes, upon the instructor to whom she is assigned, or upon her aptitudes as indicated by her choice of major.

Section 2

Those students who have had more than 3 grades of A, apiece, have been selected for special attention. Of the 286 failures, there were just 6 students with more than 3 A's each. They constitute an interesting group. Details follow.

Case 1. A student with 12 A's. A personal interview with this student revealed no bitterness whatever on the part of the student. When she took Mathematics 1 for the first time, her class met late in the afternoon, from three to four, and on one day each week from four to five. During the term it was necessary for her to accompany someone to the hospital at a time which coincided with the time set for her mathematics. She was absent from her final examination. She was a lower freshman at the time and not familiar with the regulations concerning absentee examinations. Upon repeating the course, the student made A.

Case 2. A student with 7 A's. This student had 4 A's in German, and 3 A's in French. A more detailed study of the student's official record revealed the fact that she was born in France. In recording her F in mathematics, her instructor made the significant comment, "Language Difficulty."

Case 3. A student with 6 A's. The F in Mathematics 1 for this student was recorded as "F drop."

Case 4. A student with 5 A's. This student had 2 A's in German, 1 A in Latin, 1 A in Music, and 1 A in Physical Training. Next to her F in Mathematics 1, her instructor made the comment, "Term Grade 59,[6] Final Examination very poor."

Case 5. A student with 4 A's. This student received all 4 A's in Greek. The student was born in Greece.

Case 6. A student with 4 A's. This student had 2 A's in Chemistry, 1 A in Physics, and 1 A in German. A personal interview with this student brought forth the statements that she likes mathematics, and plans to take additional courses in mathematics. The student did admit that, although she considered the F entirely fair, she was just a little bit surprised when she learned that she had received F in Mathematics 1.

It seems almost incredible that the number of the total 286 failures with more than 3 A's each should be just 6. For one must remember that the survey includes subjects such as Physical Training, Voice Culture, and Sewing, in addition to the more formal courses of the curriculum. The above case studies suggest that 3 or 4 of these 6 failures are "failures by avoidable accident,"—F's which are technical rather than F's which designate failure in study and accomplishment in the course. The remaining pitifully small number, 2 or 3, among which one would look for the proverbial student who is good in every subject except mathematics, fail one completely in the search for evidence of the existence of such a person. In Section 3 further comments are made about the A's earned by the total 286 failures.

Section 3

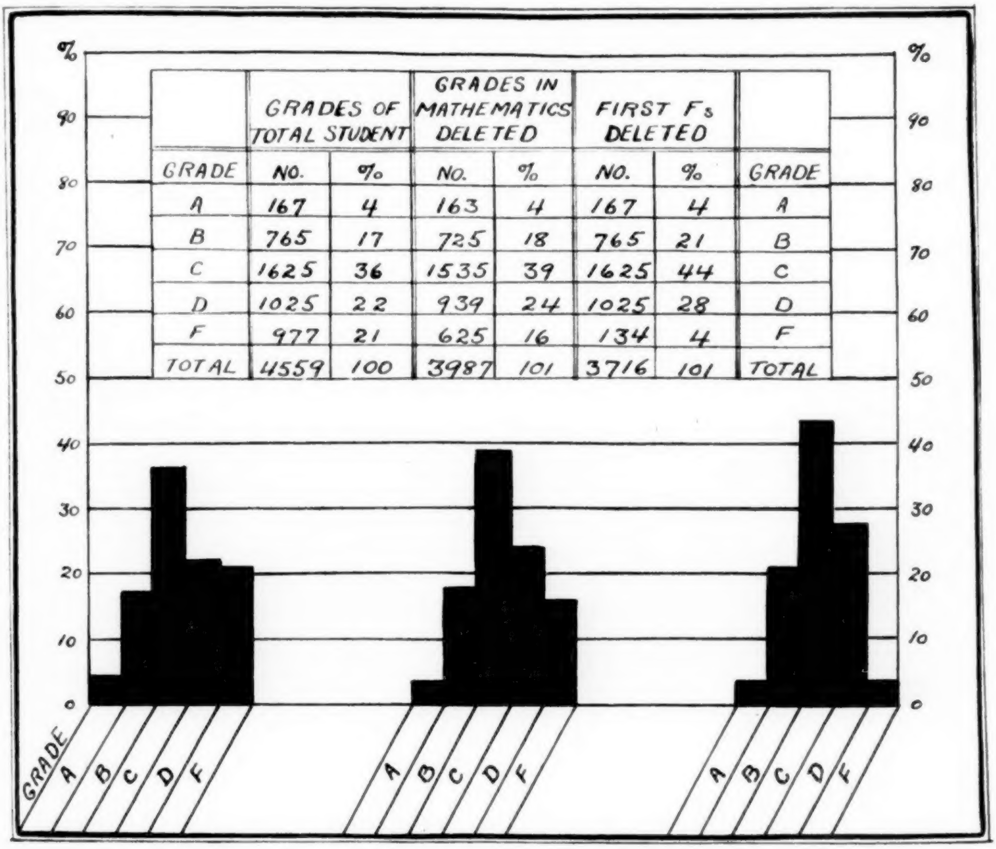
The last section of this paper presents an analysis of all the 4559 grades made by all the 286 failures.

Attention is first directed at the A's. Of the total 4559 grades, there were 167 grades of A, 17 of which were on a second or third trial, that is after one or two consecutive failures, in a particular course. It was found that the 150 first A's were distributed among 79 of the 286 students. Thus 72 per cent of the original 286 did not receive a single first A. A distribution of the 150 first A's by subjects shows that 16 were in Opera or Voice Culture (each a one half credit optional course in the Department of Music); 11 were in one credit courses in Physical Training; 3 in Sewing; and the remaining 120 were distributed among 16 other departments. In general one may conclude that the distribution of A's among both students and subjects is so wide that there is no central

[6] At Hunter College, the passing mark is 60.

tendency. The only conclusion of any value which presents itself is that 72 per cent of the failures, in about two years time, have not received a single first A.

At the other end of the scale are some interesting conclusions from a consideration of F's. Of the total 286, 50 failed in no subject other than mathematics. Of these 50, 3 received "F drop";[7] 5 received F because of absence;[8] and 7 had not had Intermediate Algebra, a course which is regarded as a prerequisite for Mathematics 1. The remaining 35, who received unqualified F's in Mathematics 1, represent only 12 per cent of 286. Regarded from another point of view, it may be said that 83 per cent of the total number of failures in Mathematics 1 received F in at least one subject in addition to mathematics.



	GRADES OF TOTAL STUDENT		GRADES IN MATHEMATICS DELETED		FIRST Fs DELETED		
GRADE	NO.	%	NO.	%	NO.	%	GRADE
A	167	4	163	4	167	4	A
B	765	17	725	18	765	21	B
C	1625	36	1535	39	1625	44	C
D	1025	22	939	24	1025	28	D
F	977	21	625	16	134	4	F
TOTAL	4559	100	3987	101	3716	101	TOTAL

(a) (b) (c)

GRAPH 4. Distribution of the Grades of "The Total Student."

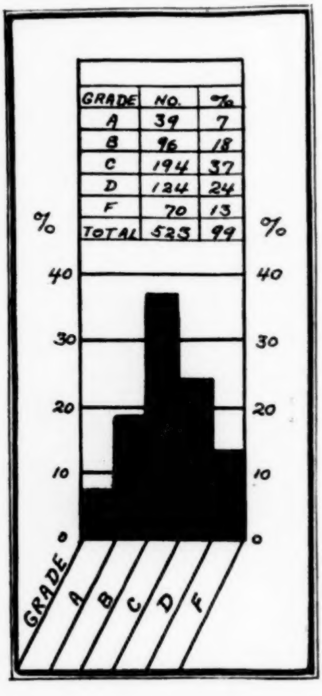
The following table shows the performance of the failures on repeated trials in Mathematics 1.

[7] "F drop" is usually the result of unofficial withdrawal from a course, on the part of a student. Such an F may or may not be due to genuine failure.

[8] At Hunter College, if a student is absent from a final examination, her grade is recorded as "F absent." If the work of the student averaged grade C or better during the term, and the Committee on Promotions is convinced that the absence was unavoidable, the student is given an absentee examination. The grade of a student, who has been absent for more than 20% of the total number of recitations during a term in a particular course, is recorded as "F absent one fifth." Her instructor has the privilege of recommending that the student receive a passing grade. The recommendation of the instructor is usually honored by the Committee on Promotions.

	A	B	C	D	F	Total
2" trial	3	36	81	70	51	241
3" trial	1	4	9	11	13	38
4" trial	—	—	—	4	2	6
Total	4	40	90	85	66	285

It is worthy of comment that the distributions on the second and third trials are far from normal.



GRAPH 5. Distribution of Major Grades of "The Total Student."

Figures concerning the length of stay at college of these failures provide interesting conclusions. At some time before September 1934, 54 of the 286 failures were discharged; 154 were dropped from college because of poor scholarship; and only 78 have remained in uninterrupted attendance at college to date. In percentages these figures are as follows,—discharged 19 per cent; dropped 54 per cent; in uninterrupted attendance 27 per cent. In other words, 73 per cent of the 286 failures have been either discharged or dropped from college because of poor scholarship, at some time during the early part of their college career.

This paper ends with a discussion of distributions of all the grades of the original 286 students. The total number of A's, B's, C's, D's, and F's, made by all the failures, were obtained. Distributions of these totals are presented as though they represented grades earned by a single individual. The graphs are called distributions of the grades of "the total student."

As one would expect, Graph 4(a), the distribution of all the grades of the total student, shows a graph which is heavily skewed in the direction of the F's. The very fact that only students who received F in Mathematics 1 are considered in this paper, helps to explain the reason for the large number of F's in this diagram. In Graph 4(b) one sees the distribution of the grades of the total student which remain after all grades in mathematics have been deleted. Strange as it may seem, Graph 4(b), although not skewed to the right to so great an extent as Graph 4(a), is still heavily skewed in the direction of the F's. Finally, in Graph 4(c) an almost normal distribution appears. In Graph 4(c) are shown the grades of "the total student" which remain after all first F's in all courses have been deleted.

One final diagram, which helps substantiate the conclusions of the preceding paragraph, follows. Of the total number of failures, 182 have remained in their originally chosen majors.[9] Graph 5 represents the distribution of the major grades earned by the 182 students who have remained in their originally chosen majors.

A comparison between Graph 5 and Graph 4 shows that the distribution of

[9] The apparent discrepancy between the 182 who have remained in their originally chosen majors and the 78 who have remained in uninterrupted attendance at college, is explained by the fact that some of the 182 students were discharged from college and then readmitted at a later date.

major grades is very much like the distribution of the grades of "the total student" which remain after all grades in mathematics have been deleted. From Graph 5 one sees that a distribution of even major grades of those of the failures who are most sure of the subject they wish to pursue, does not measure up to the normal distribution.

Summary

Consider a student entering Hunter College as a lower freshman and taking Mathematics 1 for the first time. It seems true that if the student comes from Preparatory School J, Figure 1; if she is not assigned to Instructor L, Figure 2; and if her inclinations lie in the field of Music; her chances of passing Mathematics 1 on the first trial are hindered by this unfortunate set of circumstances, over which she has no control at the time of her entrance to college. On the whole, however, the preparatory school from which she comes, the instructor to whom she is assigned, and her inclinations as indicated by her choice of major, seem to have very little influence upon her chances of passing Mathematics 1.

Consider next a freshman who fails Mathematics 1 on her first trial. This investigation shows that the chances that she will receive an A on the first trial in any course at all during her first two years at college are about 1 in 30; the chances that she will receive at least one F in some subject in addition to mathematics are about 10 in 12; the chances that she will remain in uninterrupted attendance at college during her first two years are about 1 in 4; and in general, the distribution of her grades, in all her courses and even in her major courses, is apt to be below the normal distribution.

Finally, the results of this paper should convince the most obdurate sceptic that the fiction of "the person who is good in everything except mathematics" or "the person who simply cannot do mathematics" is without basis in reality.

The Relationship Between Vocabulary and Ability in First Year Algebra

By Guy E. Buckingham
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Positive relationship has been shown between the elimination of errors in the processes of addition, subtraction, multiplication, and division of monomials and the ability to solve the more complex problems in first year Algebra.[1] The relationship between silent reading ability and ability in first year Algebra has been demonstrated to be low but positive.[2] A careful interpretation of these studies leads one to conclude that other factors are present to help or hinder the learning process in beginning Algebra.

Among these factors vocabulary would seem to be important. The correlation between paragraph comprehension using easy vocabulary and validated for peculiar techniques of reading and ability in beginning Algebra is positive and low. That is, reading to predict outcomes, to follow directions, to note details, and to get the general significance of a paragraph is positively related to ability to learn first year Algebra. Since reading and vocabulary are so intimately related it follows that the setting for the learning process in a single problem may be so changed by an unknown word as to make the total problem incomprehensible to the student.

It is now a recognized procedure to teach new words in reading by introducing them in a context already known to the student. It follows that a definition of a word out of context will depend for its significance to the student on some background context either present or not present in memory. If the new words are introduced too rapidly to be assimilated into the older meaningful contexts already in the student's possession little or no significant meaning can be attached to the new words as they are presented in the almost meaningless context of Algebra. Since Algebra is quite abstract in form and meaning, and is likely to be vague to the beginner, a too difficult vocabulary would be a serious handicap in the learning process.

Words in Algebra divide themselves rather easily into non-technical words (silo), technical Mathematical terms (cube), and technical Algebraic terms (monomial). The non-technical vocabulary will depend on initial capacity to learn plus experimental application, real and vicarious. The technical Mathematical terms will depend on initial capacity plus specialized experience largely of a vicarious nature. The same is true of the technical Algebraic terms. One might speculate concerning the amount of Algebraic terminology which is actually needed by a beginner. Rather than speculate it would be wiser to attack the problem experimentally.

Many high schools use a basic text alone in the first course in Algebra. The teachers make little effort to supplement either the reading materials or the vocabulary of the text. It is assumed that an understanding of the problems as they occur in the text will materialize regardless of vocabulary or silent reading ability. The teacher complains that the students are not able to do Algebra because they are not able to read. With much vigor the Algebra teacher tells of certain obvious (to her) word meanings which the students do not know. The fact that Algebra has its own settings,

[1] Guy E. Buckingham, *Diagnostic and Remedial Teaching in First Year Algebra.* Northwestern University Contributions to Education No. 11, School of Education, Northwestern University, Evanston, Illinois.

[2] Guy E. Buckingham, *The Relation Between Silent Reading Ability and Ability in First Year Algebra.* To be published in the next issue of *The Mathematics Teacher.*

words and meanings which must be taught in parallel with problem solving does not occur to the teacher. The Meadville, Pennsylvania, High School uses the Engelhardt and Haertter Algebra in the first year as a basic text. Little, if any, vocabulary is introduced to the students beyond that found in the basic text.

In order to learn what words would confront the beginning student, and their relationship to his problems, an analysis of the vocabulary of this book was contrasted with Thorndike's first 10,000 word list. The majority of all students studying beginning Algebra that year were around fourteen years of age. It will be recalled that in the Stanford Revision of the Binet-Simon Scale Terman used 9,000 words at the fourteen year level. Of course, this would be an average vocabulary which means that one-half of the students fourteen years of age likely had vocabularies of fewer than 9,000 words. The contrast between the basic Algebra text vocabulary and the Thorndike list was made in the following manner.

1. All words in the Algebra text not found in the first 9,000 words of the Thorndike list were tabulated. A total of 49 words were found in the text which were not in the first 9,000 words of the Thorndike list.

2. These 49 words were grouped into non-technical, technical mathematical terms, and technical Algebraic terms. Technical in this sense means peculiar to some particular kind of learning. There were found 23 non-technical words, 10 technical mathematical terms, and 16 technical Algebraic terms. There was some over-lapping which could not be avoided.

3. No frequency count was made of their occurrence in the text.

4. Tests were constructed using these words in Algebra problems. Each word above the 9,000 word level was placed in an Algebra problem which was composed otherwise of familiar words. By familiar words are meant those words relatively low in rank on standardized word lists. The students were instructed not to solve the problems but to show in the spaces below the problem what the words which were underscored meant. They were to write, draw or use any device they cared to use in making themselves clear. Following are sample items from the three divisions of vocabulary preceded by directions for taking the test.

DIRECTIONS

Write a sentence, or give another word or words, explaining what the *italicized* word means in the following problems. Use any way you desire to make your meaning clear. Give sample problems if you like. Do this in the space after each problem. *You are* NOT *to solve the problems.*

Part I

A painter can paint a house in 5 days; his *apprentice*, in 7 days. How long will it take them to paint the house by working together?

In making *castings*, $2\frac{1}{2}\%$ of the metal is lost in the melting. How much metal is needed to make a casting weighing 78 pounds?

Part II

The difference between the length and width of a rectangle is 7 feet and the *diagonal* is 13 feet. Find the dimensions.

What is the area of an *equilateral* triangle whose side is 1 foot in length?

Part III

Multiply the following *binomials:* $(2X-3)(3X-2)$.

Raise the following terms to the *power* indicated: 7^2, 5^6, $(32ab)^5$.

5. These tests were administered to 139 Meadville, Pennsylvania, High School students immediately after they had finished their first year of Algebra. In addition, the students were given the Coöperative Algebra Test, Form 1934 to measure elementary Algebra through quadratics. The tests were scored by the laborious method of reading each item to determine whether or not the student knew what the word meant in the peculiar context of the problem where it was found.

6. The scores made on the vocabulary tests were correlated with the scores made on the Coöperative Algebra Test.

Correlations

The correlation between the Coöperative Algebra Test scores and the total vocabulary test score was $.38 \pm .05$.

Part I of the Coöperative Algebra Test is composed of problems which are so worded that knowledge of such terms as sum, product, quotient, exponent, etc., would seem to be of great value in the solution of the problems. The score on this part correlates $.40 \pm .05$ with the score on the vocabulary tests.

Part II of the Coöperative Algebra Test is composed of statement problems only. This part of the Algebra test correlated $.25 \pm .05$ with the gross scores on the vocabulary test.

Part III of the Coöperative Algebra Test deals with problems of substitution, equations—including literal numbers, graphs, radicals, fractional exponents, and quadratics of which all presuppose a knowledge of terminology. A correlation of $.33 \pm .05$ was found between this part and the scores on the vocabulary test.

The twenty most frequently missed words are found in Table I.

The mean score of the group in the Coöperative Algebra Test was 41.77 which compares very favorably with the norms of the Coöperative Test Service. The sigma of the group was 15.4 which is 1.4 lower than that of the national group.

The mean score in the vocabulary test was 30.88 with a sigma of 6.28. The curve of distribution of the scores was rather regular except a slight skew toward the higher scores.

Among the more interesting findings were two kinds of definitions. The first kind showed complete irrelevancy to the problem. Some samples of these definitions are:

1. A solid is a four sided figure.
2. A silo is a place where a farmer grinds corn.
3. Schooners are wagons or trucks.
4. Shorthand is a writing that is taught in high school.
5. A see-saw is a saw which you cut wood with.
6. A fulcrum is the blade which is rough and sharp.

The second kind of definitions may be termed borderline. They show a partial concept development which is very interesting. They also raise a doubt as to whether or not one possessing definitive concepts as rudimentary in form as these has enough idea of what is wanted to do business in Algebra.

Table I

Word	Used Correctly	Missed	Omitted	Number of Students attempting to use the word	Percentage of Students using it correctly
Castings	49	37	53	139	35
Centigrade	33	67	39	139	24
Denomination	64	42	33	139	46
Digit	34	37	68	139	24
Evaluate	82	40	17	139	60
Fahrenheit	30	81	28	139	22
Frequency	23	51	65	139	17
Velocity	95	28	16	139	68
Diagonal	93	30	16	139	67
Equation	88	33	18	139	63
Parallelogram	59	51	29	139	42
Ratio	80	26	33	139	58
Tangent	20	28	91	139	14
Variation	57	27	55	139	41
Binomial	80	32	27	139	58
Formula	89	42	8	139	64
Linear Equation	58	49	32	139	42
Monomial	71	35	33	139	51
Power	99	21	19	139	71
Quadratic	38	71	30	139	20

Following are some examples which illustrate the point:

1. A circle is an object without corners and flat.

2. A circle is a round line.

3. A parallelogram is a cone shaped angle with four square corners.

4. A parallelogram is a figure which is a dilapidated rectangle.

5. A cylinder is a circle with a height.

6. A cylinder is a circle with volume.

7. A cylinder is a tall circle.

Findings

1. A significant relationship was found between the student's knowledge of words above the mean mental age level of their vocabularies and their abilities in doing problems in Algebra which depend largely on a follow-directions-type of vocabulary. That is, a good knowledge of words found above the mean vocabulary for a mental age of fourteen years of the group is associated with higher results in Algebra.

2. Some relationship to a less significant degree was found between the students' knowledge of words above the fourteen year mean mental age level and abilities to solve the so-called statement problem in Algebra.

3. The relationship between vocabulary ability above their mean mental age levels and ability to solve Algebraic problems involving substitution, equations, graphs, radicals, fractional exponents and quadratics was significant.

4. Total scores in vocabulary and total scores in Algebra showed a significant correlation.

5. As indicated by the mean and sigma scores on the Coöperative Algebra Test the group used in the experiment was better than the national average group.

6. Since the vocabulary test was locally made and administered, no national norms for comparison are available.

Interpretations

1. As indicated by the findings of this study vocabulary of an advanced degree of difficulty is one problem of the teacher of beginning Algebra. A correlation coefficient of .38 to .40 has very little significance if used alone. However, used as a measure of one factor among several factors it indicated enough relationship to be considered.

2. Vocabulary peculiar to Algebra is one type to be developed.

3. Vocabulary peculiar to Mathematics in general needs attention.

4. Vocabulary of a general nature belonging to the settings of problems plays a significant role in the solutions of problems in Algebra.

5. As indicated by the word definitions the students are in a developmental process in which the word images may be in one of the following three levels:

A. Completely undeveloped.

B. Concretely clear but abstractly not completely developed as "a cylinder is a circle with height."

C. Completely developed abstractly with concomitantly concrete imagery as in the case where the student defines and illustrates graphically.

6. It is probable that the student needs practice in abundance in interpreting the meanings of problems without completely solving them. Complete solutions place too much premium on temporary understandings with too little "total concept" drill. Limited time prevents enough "total concept" drill if each problem is carried through to a complete solution.

The frankest of men, someone asked President Eliot if he had ever been called a liar. He replied: "Yes, and the worst of it is they have proved it." He encouraged his faculty to express their real convictions even when they ran counter to his own, though he insisted that they be based on evidence rather than prejudice.

The Development of the Thermometer

By Mattie Bell Fretwell

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The invention of the thermometer must be attributed to Galileo, who in about the year 1593 made an open-air thermoscope. He never mentions this fact in any of his writings; but since Castelli, one of Galileo's ablest pupils, describes in great detail a thermoscope which Galileo showed him in 1603; and since another student of his, Viviani, reports the invention of this instrument by Galileo shortly after his arrival at the University of Padua in 1592, there seems to be no valid reason for doubting the statement that Galileo invented such an instrument, which is taken from a letter of Castelli's written in 1638.

The expansion of air—in any event the rising of heated air over a hot body—must have been one of the earliest effects of heat observed. It is therefore quite natural that this phenomenon—the expansion of air—should form the basis of the earliest thermoscopes. All the early thermometers contained air, and the stem was arbitrarily graduated.

Galileo's first thermometer consisted of a glass bulb of the size of a hen's egg, with a long stem of the thickness of a straw dipping into water, which was made to rise part way up the tube by previous warming of the bulb. It was affected, of course, by fluctuations of atmospheric pressure as well as of temperature, and was really a baroscope and a thermoscope. Being affected by changes in atmospheric pressure, this air thermometer was very imperfect.

This thermometer was used by Galileo for various purposes, such as studying freezing mixtures and recording atmospheric temperatures. It was later used (1611) by Sanctorius in the diagnosis of fevers. The word "thermoscope" was used by Bianconi in 1617, and "thermometer" in 1624 by Leurechon in *Récréation mathématique:* "Thermomètre ou instrument pour mésurer les degrez de chalour ou de froidure que sont en l'air."

The first improvement in the air-thermometer was introduced about 1632 by the French physician, Jean Rey, who simply inverted Galileo's instrument, filling the bulb with water and the stem with air. Thus water was made the thermometric substance. As Rey did not close up the upper end of the stem, there was constant danger of errors from evaporation of the water.

About twenty-five years after Rey's innovation, the Florentine Academy made the earliest sealed-in, liquid-in-glass thermometers of which there is any definite record. These academicians were pupils of Galileo, and made the Accademia del Cimento (1657–1667) famous. The instrument they made was similar in form to the ordinary thermometers now in use. That is, it consisted of a glass bulb joined to a graduated tube, the bulb and part of the tube being filled with a liquid. The peculiarity of their thermometer consisted in this, that they attempted to graduate the tube in such a way that the volume between two marks of graduation was a fixed fractional part, generally one-thousandth, of the volume of the bulb. The instrument thus made was filled with alcohol and the degrees were marked with small beads of enamel fused on the stem.

All that was needed to turn these Florentine thermometers—which were veritable *objets d'art* so far as glass work is concerned—into modern instruments was a scale that could be duplicated by independent observers and thus make measurements in different places and at different times strictly comparable. In order to fulfill this condition, it was necessary to select a fixed point or standard temperature as the zero or starting point of the graduations. Instead of making each de-

gree a given fraction of the volume of the bulb which would be difficult in practice, and would give different values for the degree with different liquids, it was soon found to be preferable to take two fixed points, and to divide the interval between them into the same number of degrees.

The problem of selecting two fixed temperatures for the thermometer and of subdividing the interval into a suitable number of degrees was taken up by the Accademia del Cimento. Following the example of the philosophers and physicians, they chose as fixed points the cold of winter and the heat of summer, dividing the intervening space into 80 or 40 equal spaces. To determine more accurately the position of these points, they defined the one to be the temperature of snow or ice in the severest frost, and the other to be the temperature in the bodies of cows and deer.

The fixed points chosen by the Florentine Academy did not prove satisfactory, and all kinds of improvements were suggested. Robert Hooke had found the melting point of ice to be very constant and proposed this in 1664 as a standard temperature. Huygens, the following year, made the same proposal for the boiling point of water, but neither of these men appears to have realized that two fixed points are an absolute necessity if the degrees of temperature are to be definitely defined. The two fixed points appear to have been introduced in 1688 by Dalancé who adopted the melting point of ice and butter as the two temperatures in question, and used linseed oil as a thermometric substance.

The thermometer invented by Sir Isaac Newton, described in the Philosophical Transactions for 1701, was a tube filled with linseed oil, and the starting point of the scale was the temperature of the human body, which Newton called twelve. He divided the spaces between his datum and the freezing point of water into 12 equal parts, and stated that the boiling point of water would be about 30 of these degrees on the scale.

In 1702 Guillaume Amontons made an improvement of Galileo's air thermometer. His air thermometer was of constant volume and consisted of a U-shaped tube with the shorter arm ending in a bulb and the longer arm measuring 45 inches. Degrees of temperature were indicated by the height in inches of the mercury column in the longer arm necessary to keep the volume constant. Amontons chose the boiling point of water as a fixed point, but being unaware of the dependence of the boiling point upon air pressure, he could not attain extreme accuracy.

It was Amontons who first arrived at the notion of absolute temperature. "It appears," he says, "that the extreme cold of this thermometer is that which would reduce the air by its spring to sustain no load at all, which would be a degree of cold much more considerable than what is esteemed very cold." From his data the absolute zero in the centigrade scale is −239.5°. Lambert, who repeated Amontons' experiments with greater accuracy, obtained data yielding −270.3°. The value now accepted is −273.1°. Lambert says, "Now a degree of heat equal to 0 is really what may be called absolute cold. Hence at absolute cold the volume of the air is 0, or as good as 0. That is to say, at absolute cold the air falls together so completely that its parts absolutely touch, that it becomes so to speak water-proof."

Stimulated by Amontons' researches Gabriel Daniel Fahrenheit (1686-1736) began to study the accurate construction of thermometers. He was greatly interested in Amontons' observations of the constancy of the boiling point. Curious to know how other liquids would behave, he made a series of tests and found that, like water, each kind had a fixed boiling point. Later he noticed that the boiling point varied with a change in atmospheric pressure. Attention to this fact contributed vastly towards exact thermometry. Ismaël Boullian constructed in 1659 a thermometer in which mercury was used for

the first time (so far as is known) as a thermometric substance, but it is Fahrenheit who deserves great credit for first bringing about the general use of mercury in thermometers.

Fahrenheit made two kinds of thermometers—the one filled with spirit of wine, the other with mercury. From his first paper published in 1724, it appears that the two fixed points chosen for the thermometers he then used where an ice-water-salt mixture and the blood temperature of the human body, and that the interval between them was divided into 96 parts. From his second paper of 1724, it seems that he used also a third point, determined by an ice-water mixture. In that paper Fahrenheit says: "The scale of those thermometers, which are used only in meteorological observations, begins with 0 and ends with 96. This scale depends upon the determination of three fixed points, which are obtained as follows: the first, the lowest, . . . is found by a mixture of ice, water, and salammoniac or sea salt, if the thermometer is dipped in this mixture then the liquid falls to the point marked 0. This experiment succeeds better in winter than summer. The second point is obtained, if water and ice are mixed without the salt just mentioned; if the thermometer is dipped in this mixture it will stand at 32° . . .; the third point is at the 96th degree, and the alcohol expands at that point if the thermometer be held in the mouth or armpit of a healthy person."

In his fifth paper he says: "In my account of experiments on the boiling point of several liquids, I mentioned that at that time the boiling point of water was found to be 212°; later I recognized through various observations and experiments that this point is fixed for one and the same weight of the atmosphere, but that for different weights of the atmosphere, it may vary either way." From this it appears that in 1724 the number 212 was not prearranged; boiling water simply happened to raise the mercury column to that point. If the interpretation of Fahrenheit's papers of 1724 is correct, then it was equally a matter of chance that 32° came to mark the freezing temperature of water, and that 180 came to stand for the number of degrees between the freezing and boiling points of water. There seems to be no direct and reliable information that Fahrenheit profited by the results of his experimentation, and discarded the two fixed points mentioned in his paper and chose the freezing and boiling temperatures of water as more convenient fixed points. A description in the *Acta eruditorum* (1714) of two thermometers which Fahrenheit gave to Christian Wolf indicates that the interval from ice-water-salt to blood temperature was first subdivided into 24 parts, then each of these into four smaller subdivisions, making 96 parts in all.

In France Réaumur was not aware of Fahrenheit's achievements. Dissatisfied with Amontons' air thermometer and strongly opposed to the use of mercury on account of its small coefficient of expansion, he attempted to make thermometers with spirit of wine which should be convenient and yet reach the desired degree of accuracy. His experiments accidently led to the observation of the contraction in volume which may result on the mixing of liquids. He found that spirit of wine mixed with one-fifth water expanded between the freezing and the boiling temperatures of water from 1000 to 1080 volume; so he divided the intervening distance on the stem into 80 parts. It was about 1730 that Réaumur introduced the thermometric scale in which the freezing point of water is 0° and the boiling point 80°.

Jean André Deluc (1727–1817) of Geneva returned to the use of mercury and emphasized its advantages with very forceful arguments.

Micheli du Crest, another scientist of Geneva, had no use for mercury. Rejecting the temperature of freezing water as a fixed point, he chose the temperature of

the earth as determined in the cellar at the Paris Observatory, 84 feet deep. He divided the interval between this and the boiling point into 100 parts and thereby obtained degrees agreeing closely with Réaumur's.

Andreas Celsius, a Swedish astronomer, in 1742 marked the boiling point 0° and the freezing point 100°. The inverted scale, with the freezing point 0°, was introduced eight years later by Märten Strömer, a colleague of Celsius. The final form of the modern centigrade scale is therefore not that of Celsius, but that of Strömer. The centigrade thermometer is often called the Celsius thermometer, as other thermometers are named after Fahrenheit and Réaumur. Strömer and Celsius may have been prompted to make their improvements in thermometry by the botanist Linné, who once wrote in a letter, "I was the first who planned to make our thermometers in which 0 is the freezing point and 100 the degree of boiling water."

Of the many different thermometric scales in actual use in the eighteenth century, all but three have passed into oblivion. In England and America the Fahrenheit scale predominates; in Germany the Réaumur; in France the Celsius.

The Magic Clock

By Royal V. Heath
New York City

Puzzle problems in elementary algebra generally revolve around the so-called "guessing" of a number.—You are asked to guess a number, then you are told to double it, triple it, or quadruple it, add some other number, subtract another one and so on until you begin to scratch your head.—Then, after you are practically off-guard—you are told what number you guessed.

Usually, such tricks are based on simple formulas so constructed that all operations necessarily lead to the original number.

Number tricks are interesting, but in order that they have pedagogical value they should contain some other elements besides the formula.—For instance, certain numbers possess some properties which are not only interesting but will be used by a pupil long after he has completed elementary algebra.—And to learn these properties is very important. They could be learned in an amusing way.

"The Magic Clock" presents the properties of arithmetic progressions and also some numbers connected with a right triangle. On the outer rim of the clock are two arithmetic progressions.

$$3,\ 6,\ 9,\ 12,\ 15, \ldots$$

$$4,\ 8,\ 12,\ 16,\ 20, \ldots$$

The difference between the two numbers located at any one position on the outer rim (4, 3), (8, 6), (12, 9), . . . is equal to the number of the hours there. And when that difference is added to the higher of the two numbers used, it will be found to be equal to the number of minutes at that location. Also the sum of the squares of any two numbers located at any one position on the outer rim is equal to the square of the number of minutes there, because $3^2+4^2=5^2$ and $6^2+8^2=10^2$, etc.

Thus the child may construct his own dial starting with the above two progressions.

THE ART OF TEACHING

A NEW DEPARTMENT

The Harbor Learned Through Mathematics

By NELL SCOTT, *John Burroughs Junior High School Los Angeles, California*

THE date for the observance of National Foreign Trade Week had been announced. In reply to a letter addressed to the Los Angeles Harbor Commission we received their last Annual Report, 1934–1935; their previous Annual Report, 1933–1934; and their Monthly Statistical Report for June, 1935. These Reports were placed in the hands of two seventh grade mathematics classes, in a junior high school, for the study of domestic and foreign trade over the wharves of the Los Angeles Harbor.

As the first step in the study, certain members of the class read aloud the main articles on the history and facilities of this harbor, and on the progress of the leading industries of this Coast with the ratings of their chief commodities of trade. Then the books were passed around in order that all the pupils might see the interesting pictures of the facilities and of the cargoes loading and unloading.

The classes, next, made a study of the terms used in expressing the various measurements. There was mention of linear feet for water frontage, of square feet for areas of terminal buildings, of acres of land for certain industries, and of cubic yards of different kinds of rock for the breakwater. In the "Annual Reports," items of commodities were expressed as so many barrels of oil, board feet of lumber, cases of canned fish, bales of cotton, rolls of news-print, boxes of fruit, and so forth; whereas, in the Monthly Statistical Report, all items were listed in terms of pounds, with some of the totals figured in tons. Following this study of terms, practice in the reading of large numbers was found to be worth while.

Then, the pupils studied the ways in which comparisons were presented in the Reports; by ratios in simple fractional or per cent form, by tables, charts, and graphs. Per cents of increase or decrease in pound-weights and in dollar-values had been computed for some of the tables of comparisons. Per cents were further used in charts; such as the chart showing that the decrease in tonnage of general merchandise over the wharves of this Port had been offset by increase in value, during the year just closed. Hence, the pupils learned that both quality and quantity must be compared in order to arrive at a true report on commerce.

The study of these comparisons was not too difficult for pupils of average seventh grade ability because the per cents had been rounded off to the nearest hundredth, and because the fractions were simple. To illustrate the type of fractions used; "In 1934–1935 the general cargo in tonnage averaged about $\frac{1}{4}$ foreign and $\frac{3}{4}$ domestic; $\frac{3}{4}$ of this classification was inbound and $\frac{1}{4}$ outbound. In value of total commerce about $\frac{1}{5}$ was foreign trade and $\frac{4}{5}$ domestic; $\frac{2}{3}$ of this value was inbound and $\frac{1}{3}$ outbound."

Interpretations of the different charts and graphs pictured in the Reports stimulated interest and a desire on the part of the pupils to attempt the drawing of graphs of trade. The last Annual Report had a chart on the "Number of Vessels Entering the Port from 1900 to 1935." A table on the "Number of Vessels by Flags Entering the Harbor in 1934–1935" furnished material for bar graphs to be drawn by some of the pupils. One pupil

drew, in the margin of her graph, the flags of the different countries represented.

To show various comparisons of Outbound and Inbound Commerce the pupils drew both bar and circle graphs. The most interesting bar graph dealt with the eleven countries of the world which carried on the heaviest trade through this Port in the year just closed. The bar for Japan was surprisingly high. Everyone was interested in knowing what commodities made up our trade with Japan. A pupil volunteered to copy, for the bulletin board, a complete list of the exports to and imports from Japan, as of the Monthly Statistical Report for June, 1935.

The boys drew graphs picturing various facts about petroleum, the greatest single item in bulk cargo at this Harbor. They found a decline in the total shipment of petroleum during this last year. They read that the depression, labor troubles, and increased shipments from shoreside facilities were contributing factors to this decline. There was an increase, however, in foreign export of petroleum, due largely to Japan's heavy demand for fuel oil. The boys were especially interested, again, in the facts about the next largest single item, the lumber. They learned that more expensive lumber and more hardwoods from all parts of the world had been received during this last year than during any previous year.

The girls traced through the Monthly Statistical Report for the travel stories of other kinds of commodities. There was the story of California fresh oranges exported during the month of June, 1935; how much value and how many pounds outbound to what different countries and to what different ports. The girls figured out that the United Kingdom, the Netherlands, Belgium, France, and Sweden were the leading markets, in that month, for oranges from this Port. The travel stories of cheese, rice, silk, cotton and rugs were also considered interesting both as to facts and as to figures.

During National Foreign Trade Week some pupils attended the Harbor Day Celebration, and, then, gave us reports of their trip. Other brought in clippings from the newspapers on the subject of Foreign Trade and of better International Good-Will.

A letter, selected as the best among several written by the pupils, was sent to the Harbor Commission thanking them for the help received through their Reports. Following are some statements quoted from the letters:

"It interests me to see your Statistics and learn other ways of using Arithmetic than just in school books."

"We have been studying percentage and graphs, so your books were of great interest and help in finding new problems for our mathematics class."

"Your maps, graphs, and articles have given us a greater knowledge of our Harbor, and the concerns that help its progress."

EDITORIALS

Facing the Facts

THE ARTICLE by Professor Douglass in this issue is very timely and coming from a Professor of Secondary Education who is friendly toward mathematics ought to carry more weight than if it came from a teacher of that subject. The same is true of the article in this issue by Mr. Klopp. For some reason or other, the attacks on mathematics have been particularly numerous recently and one can only conjecture what the final outcome will be. Professor Douglass seems to think that it is not too late for those of us who still feel that the educational value of mathematics is important to proceed to do something about it. It is to be hoped that at the next meeting of the National Council of Teachers of Mathematics in Chicago, the members will try to discuss many of these pressing problems with each other both privately and in some of the open meetings.

There is no question but that there is considerable apathy among all secondary teachers with reference to the needs for a more dynamic and helpful curriculum. Moreover, many teachers in the secondary schools are not yet duly interested in trying to find out what a general education ought to be and may find themselves ultimately in a very precarious position. It is doubtful whether mathematics teachers are any worse than teachers in the other great fields but even if they are not, they cannot console themselves by any such knowledge. It seems certain that all of us who are really interested in keeping mathematics on the high plane that it once had should take particular interest at this time not only in what mathematics may still be required in secondary education but also what mathematics above the required part may still be made useful and necessary by proper reorganization.

Dr. Slaught's Birthday

PRESIDENT Martha Hildebrandt of the National Council of Teachers of Mathematics has planned to have some recognition of Dr. H. E. Slaught's birthday at the annual banquet on Saturday evening, February 20, at the Palmer House in Chicago. It is particularly fitting that the National Council should do honor to a man who for so long has not only been an inspiring teacher for thousands of students but who also has taken an unusually wide interest in the more general sphere of mathematical education in the secondary schools of the United States and particularly in the work of the National Council. Professor Slaught has also been a guiding spirit in the Mathematical Association of America and, in fact, in all movements toward the improvement of mathematical education. It is a great source of joy to all of us that he is still with us to participate at least in spirit if not in person with all of our attempts for the improvement of mathematical instruction in the schools. Those of us who have been fortunate enough to be in his classes when he was actively engaged in teaching at the University of Chicago will feel a particular thrill in honoring a teacher whose inspiration and guidance has meant so much to us in our subsequent educational careers. *The Mathematics Teacher* wishes him many happy returns of the day.

IN OTHER PERIODICALS

By Nathan Lazar
Alexander Hamilton High School, Brooklyn, New York

Algebra and Analytic Geometry

1. Aude, H. T. R. *The use of matrices in college mathematics*. National Mathematics Magazine. 11:95–104. November 1936.

The use of matrices in connection with certain topics in algebra is well known. The author believes that in analytic geometry too, there is an especially favorable opportunity for their early introduction. By means of specific examples he illustrates the use of matrices not only as symbols of notation, but also as operators for the solution of problems of the following types:

a. matrices for points and lines
b. matrices for the equations of second degree in the plane
c. polarization by matrices
d. quadratic surfaces
e. the one dimensional quadratic form.

2. Colwell, R. C. *Mechanical devices for drawing Lissajou's figures*. School Science and Mathematics. 36: 1005–1006. December 1936.

With the help of two diagrams the writer describes a machine invented by Mr. Fullner, connected with the physics laboratory at the West Virginia University. The device "will draw sine, cosine, and straight line curves with simple combinations of the three."

3. Hatch, Hollis D. *Horner's method—shortened*. School Science and Mathematics. 36: 1007–1008. December 1936.

The author describes a method for getting an approximation of the root of the depressed equation closer than the one that is usually obtained by dropping all powers of x, except the linear one.

4. Read, Cecil B. *Logarithms versus cologarithms*. School Science and Mathematics. 36: 981–85. December 1936.

The author reports on an unpublished Doctor's field study made "in an effort to determine the relative efficiency of computation by the use of logarithms alone as contrasted with computation by use of logarithms supplemented by cologarithms." After a careful exposition of the technique and materials used in the study, he makes the following recommendations:

a. "The subject of cologarithms should not be required.
b. "When there is abundance of time the use of cologarithms may be introduced as an optional method.
c. "No definite statements should be made regarding the relative advantages or disadvantages of computation by either method."

Geometry

1. Hope-Jones, W. *The rhombic dodecahedron for the young*. The Mathematical Gazette. 20: 254–57. October 1936.

A description of a method for interesting youngsters of 13 or 14 in the treasures of the rhombic dodecahedron, by starting with the study of a bee's cell. Diagrams, directions, and exact dimensions are included.

2. Kramer, Edna E. *Honors courses in mathematics*. High Points. vol. 18, no. 10. December 1936. pp. 42–3.

The author explains the method described below that is used in the Thomas Jefferson High School to encourage those students who show unusual ability and interest in the study of mathematics. Before the beginning of the summer vacation a syllabus is offered to the students who have received 90% or higher in tenth year mathematics. The syllabus contains a list of those theorems and difficult originals that are omitted in ordinary class work. The students are asked to study the theorems and to prepare a notebook containing the proofs of the exercises. Upon return to school in September, a test is given covering the work of the syllabus. Those who submit creditable papers receive an "honor certificate," while those with perfect marks are mentioned in the school paper. It is interesting to note that, although no school credit is offered for the above work, 150 students nevertheless applied for the privilege, and of that number 30 carried the initial ambition to a successful culmination.

3. McCarthy, J. P. *Huygens' proof of the Theorem of Pythagoras*. The Mathematical Gazette. 20: 280–81. October 1936.

An exposition of a purely geometric proof by a famous physicist of the well-known theorem.

4. Musselman, J. R. *The triangle bordered with squares.* The American Mathematical Monthly. 43: 539–48. November 1936.

After a brief survey of the literature on the above topic the writer states that it is the purpose of his paper "to show how some of these results may be easily proved by an analytic method, and to use this method to obtain additional properties of the figure."

Miscellaneous

1. Alexander, Jerome. *Mathematical imagery and physical phenomena.* Scripta Mathematica. 4: 139–45. April 1936.

An acute analysis of the reciprocal effects that advances in the natural sciences and mathematics tend to have upon each other.

2. Dunnington, G. Waldo. *Oslo under the integral sign.* National Mathematics Magazine. 11: 85–94. November 1936.

An interesting account of the International Congress of Mathematicians that was held at Oslo, Norway, July 13–16, 1936. Three full-page photographs are included.

3. Ettlinger, H. J. *Mathematics and the hypotheses of science.* National Mathematics Magazine. 11: 71–77. November 1936.

By means of many examples, the author proves that mathematics and the natural sciences tend to exert mutual influences upon each other. While "numerous examples may be cited where the needs of science have led to striking advances in mathematics . . . one may point out the more recent profound influence of the concept of pure mathematics on the whole outlook of the physical sciences."

4. Keyser, Cassius Jackson. *Panthetics.* Scripta Mathematica. 4: 126–38. April 1936.

A report of an address given at the Galois Institute of Mathematics, May 23, 1936. In his usual lucid and entertaining style the author discusses the following logical and philosophic problems:

a. Wonder or curiosity. Questions and pseudo-questions.
b. Answers and propositions. Categorical and hypothetical, or implicatory.
c. Established proposition. Scientific proposition and mathematical proposition. Science and mathematics.

5. Morton, R. L. and Miller, Leslie Haynes. *A comparative study of the scholarship records of students who major in mathematics.* School Science and Mathematics. 36: 965–67. December 1936.

The writers made a detailed study of the records of the 2,262 persons who were graduated from Ohio University in the five-year period beginning January 1, 1931, and ending December 31, 1935. After a detailed exposition of the technique employed, they arrived at the following conclusion:

" . . . the odds are 2,800 to 1 that those who major in mathematics are superior scholastically to all other graduates. Mathematics is not a popular subject. In the five-year period reviewed here only 63 persons graduated with majors in this subject. This is less than three per cent of the total number of baccalaureate graduates. But those who elect mathematics as a major are, generally speaking, able to make relatively high scholastic marks."

6. Neugebauer, Otto. *The history of mathematics.* National Mathematics Magazine. 11: 17–23. October 1936.

Penetrating comments on the difficulties inherent in the writing of a history of mathematics. " . . . every single historical investigation can count as a usable *preliminary* performance toward further synthesis only if it is guided by two viewpoints: to see the history of mathematics in the framework of general history, and to understand mathematics itself not as a collection of formulas to be continually increased, but as a living unity."

7. Seidlin, Joseph. *The place of mathematics and its teaching in the schools of this country.* National Mathematics Magazine. (a) 10: 304–07. May 1936. (b) 11: 24–45. October, 1936.

The author tabulates the replies given to the questions that he sent out to 1,000 teachers of college mathematics in 520 colleges and universities. He also reproduces 132 extracts from the comments made by those who replied to his questionnaire.

The following are the questions:

I. Do you believe that at least one year of algebra and one year of geometry should be required of all high school pupils:
 (a) As now taught?
 (b) Changed (presumably improved) in content?

II. Do you believe that at least six semester hours of mathematics should be required of all students in a liberal arts college?

III. Do you believe that mathematics at the level indicated below requires "special mathematical ability?"
 (a) Ninth grade?
 (b) Tenth grade?
 (c) First year college?

IV. Do you believe that only "superior" or "scientifically-minded" students should be encouraged to elect mathematics beyond the 8th, 9th, 10th, 11th, 12th, 13th (first year college) grade? Indicate grade.

V. Comments.

NEWS NOTES

The Southern Branch of the Colorado Education Association, Mathematics Section, has applied for a charter of affiliation with the National Council. Mr. H. W. Charlesworth of Denver, state director for the National Council addressed the meeting in Pueblo concerning advantages of affiliation on Thursday, Nov. 5.

The complete program of the Denver meetings of the Mathematics Section which is affiliated with the National Council follows:

MATHEMATICS

President, Adalyn Seevers

Vice-President, Beth Irwin

Secretary-Treasurer, L. Denzil Keigley

Thursday, November 5, 2:00 P.M.

Business Session—

Report of National Council Meeting in Portland, L. Denzil Keigley, Morey Junior High School

"What Mathematics Our Employees Need" Walter Beans, Vice-President and Treasurer of Daniels and Fisher

"How Mathematics Helps Our Employees," P. E. Remington, General Auditor of Bell Telephone Company

"The Progressive Movement in Mathematics", A. E. Mallory, Professor of Mathematics, Colorado State College of Education, Greeley

Discussion

Friday, November 6

Mathematics Section Luncheon

2:00 P.M.

Address—"Is There a Psychology of Mathematics?", Dr. Frederic B. Knight, Professor of Education and Psychology, State University of Iowa

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Officers of the Colorado Branch of the National Council elected for the ensuing year are: Miss Adalyn L. Seevers, Fort Morgan, Colo., President; Mrs. Madeline Strang, Colorado Springs, Colo., Vice-President; Mr. L. Denzil Keigley, Denver, Secretary-Treasurer.

L. Denzil Keigley of Denver was elected as official delegate from the Colorado Branch to the February meeting in Chicago of the National Council.

The afternoon sessions of the November meetings were attended by an average of 190 mathematics teachers, and the luncheon on Nov. 6 had an attendance of 94.

The executive council of the Colorado Branch of the National Council will meet in Denver on Saturday, February 6, 1937 to plan the spring meeting, which will be held early in March.

Colorado Mathematics teachers are now publishing a mathematics bulletin three times per year. The December issue will contain 12 pages. Miss Alfhild Alenius of Denver is editor.

A very interesting lecture on cryptology was presented by Professor Joseph S. Galland of Northwestern University to the Women's Mathematics Club of Chicago and Vicinity on Saturday, December 2, in the Green Room at Mandel Brothers.

Professor Galland talked on "Ciphers—Military and Literary—" and explained the use of figures in codes. He illustrated his talk with stereopticon slides, showing the practical values of these ciphers in codes of various descriptions.

The talk was particularly interesting to teachers of mathematics because it showed the demand on mathematics made by different fields.

Range Mathematics and Science Club

A meeting of the Range Mathematics and Science Clubs was held in Mountain Iron, Senior High School, on Thursday, December 3, at 6:00 P.M.

The program numbers for the joint session were:

1. Address of Welcome by Mr. O. H. Whitehead, Superintendent of Public Schools, Mountain Iron.
2. Flute duet by Miss Grace Halliday and Mr. Harry Halliday of Virginia, accompanied by Miss Elsa Perala, Mountain Iron, at the piano.
3. Address "Recent Trends in the Secondary School Curriculum" by Dr. Oliver R. Floyd, Principal, University High School.

The speaker for the Science Club was Mr. Barney Erwin, Manager, Commercial Department of Radio Station WMFG, Hibbing and

Virginia. The speakers for the Mathematics Club were:

Miss Ethel V. Nelson, Supervisor, Mountain Iron School, and Dr. L. B. Kinney, Director of Mathematics, University High School.

Miss Nelson discussed the topic, "Diagnosing Arithmetical Difficulties in the Elementary Grades." Dr. Kinney addressed the club on "Mathematical Requirements of Commercial Positions."

The committee in charge of the general arrangements consisted of: Miss Helen Johnson, Miss Esther Farrington, Miss Helmi Rahko, Mr. J. F. Muench, H. G. Tiedeman, all of Mountain Iron, and H. K. Savre of Buhl. Mr. Savre is the president of the Range Science Club, and H. G. Tiedeman heads the Range Mathematics Club.

The third meeting of the Men's Mathematics Club of Chicago and the Metropolitan area was held on Friday, Dec. 18, 1936. The program follows:

Are We Teaching What the Pupils Need?, J. A. Nyberg.

"How Are the Mathematics Teachers to Meet the Proposed Changes in the Chicago High School Curriculum?" was discussed by a Committee composed as follows: Dr. J. S. Georges, Wright Junior College, Chairman, W. G. Hendershot, Roosevelt High School, Thomas J. Nolan, Amundsen High School, Joseph A. Nyberg, Hyde Park High School, Joseph M. O'Rourke, Lane Technical High School.

OUTLINE OF POINTS FOR DISCUSSION

By DR. J. S. GEORGES

Chairman of the Committee

I. What factors are responsible for the belief that mathematics is not needed?

II. To what extent is the present lack of understanding of the true educational nature of mathematics due to

1. Present organization of algebra?
   a) Manipulative
   b) Specialization
   c) Lack of integration with life
   d) Not interpreted as a language
   e) Overlooking the fact that it is a quantitative method of thinking
   f) Not emphasizing its laws as applicable to other fields
2. Present organization of geometry?
   a) Space perception obtainable intuitively and inductively
   b) Geometric facts obtained in previous courses
   c) Deductive method appreciated by more mature minds
   d) Logical reasoning obtainable in other subjects
   e) No definite course or unit objectives
   f) Direct measurement not an objective
   g) Indirect measurement not an objective
   h) Constructional work not emphasized
   i) Time requirement too great

III. To what extent are the attacks on mathematics justified by the teaching methods employed in mathematics classes?

1. Learning products not satisfactory
   a) No carry over into the other subjects
      (1) Poor training in arithmetic
      (2) Poor training in algebra
   b) High mentality
   c) Unsatisfactory grading as to mentality
   d) Lack of motivation
   e) Lack of remedial treatment
2. Teacher training inadequate
3. Less experimentation in mathematics classes
   a) Courses stereotyped
   b) Activity programs not utilized
   c) Project method overlooked

IV. Will the liberalization of requirements for graduation prove an advantage or disadvantage for enrollment in mathematics classes?

1. At par with other subjects
2. Improve organization
3. Improve methods of instruction
4. Attract better students
5. Enhance learning
6. Raise standards
7. Improve learning products

The Detroit Mathematics Club Program for the year 1936–1937 is as follows:

November 19, 1936, Northwestern High School

"Living Mathematics for the General Pupil," Virgil S. Mallory, Professor of Mathematics Instruction, State Teachers College, Montclair, New Jersey. Tea 3:30 P.M.—Address 4:30 P.M.

February 27, 1937, Noonday Luncheon Meeting

"Quantitative Aspects of Everyday Social-Economic Problems," George A. Boyce, Specialist in Mathematics, Bronxville Public Schools, Bronxville, New York. Hour and place to be determined.

April 22, 1937, Eastern High School

"Are the Newer Ideas in Teaching of Algebra Workable," Symposium Chairman, Dr. J. G. Umstattd, Associate Professor of Secondary Education, Wayne University, Detroit. Speakers: Agnes Crow, Southeastern High School; Paula Henze, Eastern High School, Lenore Tremper, Jefferson Intermediate School; Elmer Harrington, Greusel Intermediate School; Joseph Walsh, Denby High School; Orrin Seaver, Mac Kenzie High School. Tea 3:30 P.M., Symposium 4:30 P.M.

May 20, 1937, Hutchins Intermediate School

"Correlating Mathematics With Other School Subjects," Mrs. Florence Brooks Miller, Shaker Heights High School, Shaker Heights, Ohio.

Election of Officers. Tea 3:30 P.M.—Address 4:30 P.M.

Officers: President, Mr. Enos Porter; Vice President, Miss Hildegarde Beck; Secretary, Miss Sarah Jane Smith; Treasurer, Mr. Wade O. Hulbert; Chairman of Publicity, Miss Blanche Covey.

Ten Questions Asked and Answered Concerning The National Council of Teachers of Mathematics*

By EDWIN W. SCHREIBER
Secretary, Macomb, Ill.

1. *What Is the National Council of Teachers of Mathematics?*

A national association, organized February 24, 1920, at Cleveland, Ohio, incorporated April 28, 1928, under the laws of the State of Illinois, to assist in promoting the interests of mathematics in America, especially in the elementary and secondary fields, by holding meetings, for the presentation and discussion of papers, by conducting investigations for the purpose of improving the teaching of mathematics, by the publication of papers, journals, books, and reports, thus to vitalize and coordinate the work of the many local organizations of teachers of mathematics and to bring the interests of mathematics to the attention and consideration of the educational world.

2. *How Many Members?*

About one hundred charter members in 1920, by 1923 some 3,400 members and at present (Nov. 1936) some 4,300 members, representing every state in the Union and twenty foreign countries. Any person who is interested in the field of mathematics is eligible for membership in the Council. The annual dues are $2.00.

3. *Who Are the Present Officers?*

Herbert E. Slaught, Honorary President; Martha Hildebrandt, President; Florence Brooks Miller, 1st Vice-President; Mary Kelly, 2nd Vice-President; Edwin W. Schreiber, Secretary-Treasurer; William D. Reeve, Editor-in-Chief of *The Mathematics Teacher*; Vera Sanford, Associate Editor; W. S. Schlauch, Associate Editor; and nine additional directors: William Betz, H. C. Christofferson, Edith Woolsey, M. L. Hartung, Mary A. Potter, Rolland R. Smith, E. R. Breslich, Leonard D. Haertter, Virgil S. Mallory.

4. *What is the Official Journal of the National Council?*

The Mathematics Teacher, published eight months each year, October to May inclusive. It contains papers by leading mathematicians in America as well as successful classroom teachers of mathematics. Some of the special features are: "The Art of Teaching"; "In Other Periodicals"; "New Books"; "News Notes." Its articles are catalogued in "Education Index," and are noted in several foreign journals.

5. *What Are the Yearbooks?*

In 1926 the National Council published its first yearbook on "A General Survey of Progress in the Last Twenty-five Years"; in all, eleven yearbooks have been published, the eleventh appearing in 1936, entitled: "The Place of Mathematics in Modern Education." The yearbooks may be obtained for $1.75 per volume from the Bureau of Publications, Teachers College, Columbia University, New York City. On several occasions the yearbooks have been included in the sixty best educational books of the year, a list published each year in the N.E.A. journal.

6. *What Are Affiliated Organizations?*

Any local club of mathematics teachers may become affiliated with the National Council upon passing a resolution to that effect and filling out the proper blank of affiliation supplied by the Secretary. We now have more than thirty

* Professor Schreiber and Miss Martha Hildebrandt, the president of the National Council of Teachers of Mathematics are promoting a vigorous campaign for new members in the states of Wisconsin, Illinois, Indiana and Michigan in connection with the Annual Meeting of the Council in Chicago on February 19 and 20 at the Palmer House. The material sent out by them includes a letter, a membership blank, and the ten questions and answers given in this article—THE EDITOR.

affiliated organizations from New York to California and from Louisiana to Minnesota. An engraved certificate is issued to each club without cost. There is no fee for affiliation.

7. *How Many Annual Meetings Have Been Held?*

The National Council has held seventeen annual meetings: 1st: Cleveland, 1920; 2nd: Atlantic City, 1921; 3rd: Chicago, 1922; 4th: Cleveland, 1923; 5th: Chicago, 1924; 6th: Cincinnati, 1925; 7th: Washington, 1926; 8th: Dallas, 1927; 9th: Boston, 1928; 10th: Cleveland, 1928; 11th: Atlantic City, 1930; 12th: Detroit, 1931; 13th: Washington, 1932; 14th: Minneapolis, 1933; 15th: Cleveland, 1934; 16th: Atlantic City, 1935; 17th: St. Louis, 1936.

8. *Where and When Is the Next Annual Meeting to Be Held?*

The Eighteenth Annual Meeting of the National Council will be held at the Palmer House in Chicago, February 19–20, 1937. The special features of this convention will be: "A Walking Talkie"; Significant Recognition of the 75th birthday of our Honorary President, Dr. Herbert E. Slaught; A Discussion Luncheon with the officers of the Council as Hosts; Papers and discussions by prominent leaders in the field.

9. *Does the National Council Hold Meetings Other Than Annual?*

Yes. It has held three summer meetings at the time of the N.E.A. summer meetings; Chicago, 1933; Washington, 1934 (sent a representative); Denver, 1935; Portland, 1936. The Council is planning a meeting for Detroit, 1937. We have had one winter meeting with the A.A.A.S. at Pittsburgh, 1934.

10. *What of the Future?*

The National Council has set as an objective 10,000 members, the goal to be reached in the next few years. We extend to all interested a cordial invitation to JOIN NOW.

Program of the Eighteenth Annual Meeting

of the National Council of Teachers of Mathematics
Palmer House - Chicago, Illinois

Friday Afternoon, February 19, 1937

2:45 P.M.—Trip to the Adler Planetarium—Admission $0.25

Friday Evening, February 19

8:00 P.M.—*General Meeting*—Grand Ball Room

Address of Welcome
James E. McDade, Assistant Superintendent of Schools, Chicago, Illinois

Response
Florence Brooks Miller, Shaker Heights Junior High School, Cleveland, Ohio

Mathematics and Life
Albert A. Bennett, Brown University, Providence, R. I.

Projects in High School Mathematics
Beulah I. Shoesmith, Hyde Park High School, Chicago, Illinois

Saturday Morning, February 20, 1937

8:30 A.M.—*Business Meeting*—Private Dining Room Number 14

9:30 A.M.—ARITHMETIC SECTION: Grades 1–8—Club Dining Room
Presiding: Florence Brooks Miller

1. Organization of the Program
C. L. Thiele, Director of Exact Sciences, Detroit Public Schools, Chairman
2. Significance, Meaning and Insight, These Three
Dr. B. R. Buckingham, Editorial Department, Ginn and Company
3. Teaching Pupils to Teach Themselves
Dr. H. G. Wheat, Professor of Education, West Virginia University
4. Methods and Devices for the Development of Resourcefulness
Dr. Arthur S. Otis, Psychological Editor, World Book Company

Discussion

9:30 A.M.—HIGH SCHOOL SECTION: Grades 9–12—Private Dining Room Number 14
Presiding: Mary Kelly

1. Providing for Individual Needs in Mathematics
Virgil S. Mallory, New Jersey State Teachers College, Montclair, New Jersey
2. An Experiment Dealing With Slow Learning Pupils in Mathematics
Raleigh Schorling, University of Michigan, Ann Arbor, Michigan
3. Curriculum Hints from the Night School
Theo Donnelly, West Division High School, Milwaukee, Wisconsin
4. Revealing the Vitality of Mathematics
Kate Bell, The Lewis and Clark High School, Spokane, Washington

Discussion

9:30 A.M.—JUNIOR COLLEGE SECTION—Club Lounge
Presiding: E. J. Moulton

1. Off the Beaten Path
Mayme I. Logsdon, University of Chicago, Chicago, Illinois
2. Some Problems of Junior College Mathematics
H. W. Bailey, University of Illinois, Urbana, Illinois
3. Business and Finance Mathematics in the Junior College Curriculum
W. C. Schlauch, New York University, New York City

Discussion

Saturday Noon, February 20

12:00—*Discussion Luncheon*—Red Lacquer Room
Price, $1.30 per plate

Saturday Afternoon, February 20

2:30 P.M.—*General Meeting*—Grand Ball Room

1. Mathematics and the Integrated Program
W. D. Reeve, Teachers College, Columbia University
2. A Classroom Project in Intuitive Geometry
William Betz, Specialist in Mathematics, Rochester, New York

Saturday Evening, February 20

6:30 P.M.—*Annual Banquet*—Red Lacquer Room
Price, $2.00 per plate

Recognition of Dr. H. E. Slaught's Birthday
Address—A New Deal from Old Cards
H. E. Buchanan, Tulane University, New Orleans, Louisiana

Saturday Noon—Discussion Luncheon—Red Lacquer Room
(12:00 o'clock)

Florence Brooks Miller
"What can be done to increase the interest of the class room teacher in the National Council of Teachers of Mathematics?"

Mary Kelly
"What should the Senior High School course in Mathematics include?"

Edwin W. Schreiber
"Some of the teaching problems of Plane Geometry."

Wm. D. Reeve
Significant New Developments in Mathematics

Vera Sanford
"A course for Normal School Freshmen which will impress them with the importance of mathematics."

W. S. Schlauch
"What are the requirements for a good examination in mathematics?"

Wm. Betz
"The problem of retardation in grades 7 and 8."

H. C. Christofferson
"Meaningful teaching and measuring results."

Edith Woolsey
"Consumers mathematics for the ninth grade."

M. L. Hartung
"What are some of the promising recent developments or trends in the teaching of geometry?"

Mary A. Potter
"All kinds of pictures we use in teaching mathematics."

Rolland R. Smith
"Introduction to Geometry."

Leonard D. Haertter
"What should be the contribution of mathematics to the education of children."

E. R. Breslich
"Questions relating to the improvement of teaching mathematics."

Virgil S. Mallory
"Providing challenging material for the bright pupil."

C. A. Austin
"Motivating high school mathematics."

Raleigh Schorling
"Some suggestions relating to mathematics courses for non-college groups."

John P. Everett
"What is the meaning of transfer in the very elementary phases of mathematics and does this meaning change with increasing maturity of the student?"

J. O. Hassler
"Bridging the gap between high school and college mathematics."

If you wish to attend the luncheon, please send your name, your first, second and third choice in topics and a check for $1.30 (includes state tax) to Mr. E. W. Schreiber, 719 West Adams Street, Macomb, Illinois. He will send you your ticket and the topic for which your reservation has been made. Reservations will be filled in the order in which they are received.